ST. JEAN DE BREBEUF
MATHEMATICS

CHAPTER 5.4

SOLVING PROBLEMS USING LINEAR RELATIONS

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KEY CONCEPTS

A linear system can be solved by graphing or algebraically

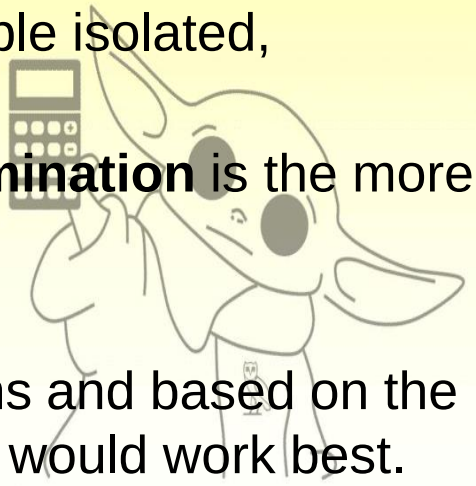
→Solving linear systems algebraically gives us the *exact* answers and the typical problems involved with graphing lines are avoided

Depending on the form of both equations, one algebraic method is typically better than the other

→ If *one* (or both) of the equations already has a variable isolated, **substitution** is the more efficient method

→ If *both* equations are in the form of Ax + By = C, **elimination** is the more efficient method

In this lesson, we will look at different types of questions and based on the form of the equations, we will determine which method would work best.



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EXAMPLE 1 *Goals*

Sara and Tara both play on the National Hockey Team

Sara has **3 times** as many goals as Tara.

The sum of Sara and Tara's goals scored is **16**.

How many goals did each player score?

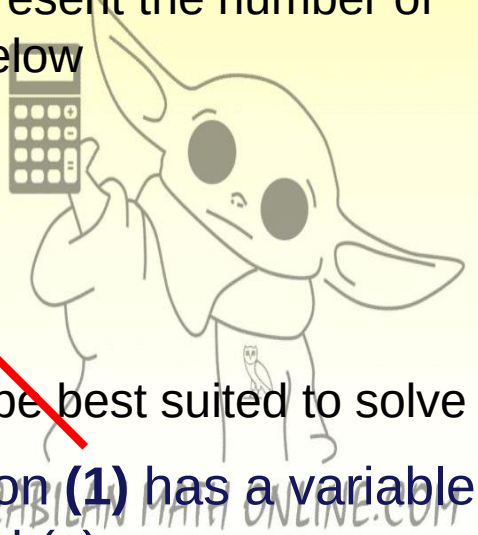
Using **s** to represent the number of **Sara's goals** and **t** to represent the number of **Tara's goals**, the linear system is defined by the equations below

Sara's goals: **(1)** $s = 3t$

Total goals: **(2)** $s + t = 16$

(a) Based on the form of the equations, which method would be best suited to solve this linear system? ___**SUBSTITUTION**___ Explain

Equation **(1)** has a variable isolated (s)



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EXAMPLE 1 *Goals*

Using **s** to represent the number of **Sara's goals** and **t** to represent the number of **Tara's goals**, the linear system is defined by the equations below

Sara's goals: **(1)** $s = 3t$

Total goals: **(2)** $s + t = 16$

Equation **(1)** has a variable isolated

Substitute **(1) s = 3t** into **(2)**

(2) $s + t = 16$

$3t + t = 16$

$\frac{4t}{4} = \frac{16}{4}$

$t = 4$

Next, we need to solve for Sara's goals by substituting **t = 4** into *any* of the equations
→Easiest to use **(1)** because it solves for **s** directly

$s = 3t$

$= 3(4)$

$= 12$

Tara has **4 goals** and Sara has **12 goals**.

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EXAMPLE 2 *How Many Cars*

Billionaire Ted collects BMWs and Mercedes-Benz cars and has a total of **10 cars**.

Next year, Ted would like to have *double* the amount of BMWs and by this time, he will have a total of **16 cars**.

How many of each car does Ted have now?

Using **b** to represent the **number of BMWs** and **m** to represent the **number of Mercedes-Benz** cars, the two equations which model this situation are

Total # of cars now: **(1)** $b + m = 10$

Total # of cars next year: **(2)** $2b + m = 16$

Using the equations above, solve the linear system to determine how many of each car Ted has now.

(a) Based on the form of the equations, which method would be best suited to solve this linear system?

ELIMINATION

Explain

Both Equations **(1)** and **(2)** are in the form $Ax + By = C$

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EXAMPLE 2 *How Many Cars*

Using **b** to represent the **number of BMWs** and **m** to represent the **number of Mercedes-Benz** cars, the two equations which model this situation are

Total # of cars now: **(1)** $b + m = 10$

Total # of cars next year: **(2)** $2b + m = 16$

(b) Using the above linear system, determine **how many of each car** Ted has today

(1) $b + m = 10$

(2) $-\ 2b + m = 16$

$-1b = -6$

$\frac{-1b}{-1} = \frac{-6}{-1}$

$b = 6$

$1b - 2b = -1b$

$10 - 16 = -6$

STEPS:

1. Choose a variable to eliminate

→ Choose a variable where <u>both</u> co-efficients are the same

2. If necessary, **multiply** the equations by a factor to make the co-efficients the same

3. **Add** or **subtract** the equations to eliminate the variable

4. Solve for the remaining variable

5. Substitute this variable into **(1)** or **(2)** to solve for the other

6. State the solution

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EXAMPLE 2 *How Many Cars*

Using **b** to represent the **number of BMWs** and **m** to represent the **number of Mercedes-Benz** cars, the two equations which model this situation are

Total # of cars now: **(1)** b + m = 10

Total # of cars next year: **(2)** 2b + m = 16

(b) Using the above linear system, determine **how many of each car** Ted has today

(1) b + m = 10
(2) − 2b + m = 16

$$-1b = -6$$
$$\frac{-1b}{-1} = \frac{-6}{-1}$$
$$b = 6$$

Sub **b = 6** into *any* equation
→ Easiest to sub into **(1)**

$$b + m = 10$$
$$6 + m = 10$$
$$m = 10 - 6$$
$$m = 4$$

STEPS:

1. Choose a variable to eliminate

→ Choose a variable where <u>both</u> co-efficients are the same

2. If necessary, **multiply** the equations by a factor to make the co-efficients the same

3. **Add** or **subtract** the equations to eliminate the variable

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Total # of cars now: **(1)** b + m = 10

Total # of cars next year: **(2)** 2b + m = 16

(b) Using the above linear system, determine **how many of each car** Ted has today

(1) b + m = 10
(2) − 2b + m = 16

$$\frac{-1b}{-1} = \frac{-6}{-1}$$

$$b = 6$$

Ted has **6 BMWs** and **4 Mercedes-Benzs**.

$$b + m = 10$$

$$6 + m = 10$$

$$m = 10 - 6$$

$$m = 4$$

STEPS:

1. Choose a variable to eliminate

→ Choose a variable where <u>both</u> co-efficients are the same

2. If necessary, **multiply** the equations by a factor to make the co-efficients the same

3. **Add** or **subtract** the equations to eliminate the variable

4. Solve for the remaining variable

5. Substitute this variable into **(1)** or **(2)** to solve for the other

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EXAMPLE 3 *Breaking Even*

"Breaking even" is when the level of production at which the costs of production is equal to the revenues for a product (there is no profit or loss).

The school athletic council is planning to sell hoodies for **$45** each.

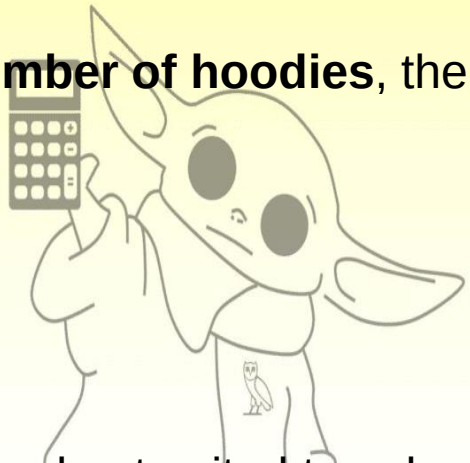
The costs to produce the hoodies is **$25 each** plus a one-time set up fee of **$7000**.

How many hoodies does the athletic council have to sell in order to break even?

Using **C** to represent the **total cost** and **h** to represent the **number of hoodies**, the linear system that models this problem are below

Selling costs: **(1)** $C = 45h$

Production costs: **(2)** $C = 25h + 7000$



(a) Based on the form of the equations, which method would be best suited to solve this linear system? **SUBSTITUTION** Explain Variable is isolated in *both* equations

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EXAMPLE 3 *Breaking Even*

Selling costs: **(1)** C = 45h

Production costs: **(2)** C = 25h + 7000

(b) Using the above linear system, determine **how many hoodies** the athletic council has to sell to break even.

We can sub Equation **(1) C = 45h** into Equation **(2)**

(2) $C = 25h + 7000$

$45h = 25h + 7000$

$45h - 25h = 7000$

$\frac{20h}{20} = \frac{7000}{20}$

$h = 350$

Gather all variables/letters to the **left**

The athletic council will need to sell **350 hoodies** in order to break even.

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EXAMPLE 4 *Buying Stuff*

Arnold is a sneakerhead and decided to buy pairs of Jordans and Yeezys.

On StockX, a pair of Jordans cost **$300** each and a pair of Yeezys cost **$500** each. Arnold spent a total of **$7600**.

Arnold purchased a total of **20 sneakers**.

How many of each sneaker did Arnold buy?

Using **j** to represent the **number of Jordans** and **y** to represent the **number of Yeezys**, the linear system that models this problem are below

Money spent: **(1)** $300j + 500y = 7600$

Total number of sneakers: **(2)** $j + y = 20$

(a) Based on the form of the equations, which method would be best suited to solve this linear system?

ELIMINATION

Explain

Both Equations **(1)** and **(2)** are in the form $Ax + By = C$

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EXAMPLE 4 *Buying Stuff*

Money spent: **(1)** $300j + 500y = 7600$

Total number of sneakers: **(2)** $j + y = 20$

(b) Using the above linear system, determine **how many of each sneaker** Arnold purchased.

(1) $300j + 500y = 7600$

300 x (2) $j + y = 20$

(1) $300j + 500y = 7600$

(2) – $300j + 300y = 6000$

We can eliminate **j** or **y**
→We will eliminate **j**

→We must multiply each term in **(2)** by **300** to make **j** to have the same co-efficient

STEPS:

1. Choose a variable to eliminate

→ Choose a variable where <u>both</u> co-efficients are the same

2. If necessary, **multiply** the equations by a factor to make the co-efficients the same

3. **Add** or **subtract** the equations to eliminate the variable

4. Solve for the remaining variable

5. Substitute this variable into **(1)** or **(2)** to solve for the other

6. State the solution

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(1) $300j + 500y = 7600$

300 x (2) $j + y = 20$

(1) $300j + 500y = 7600$

(2) **–** $300j + 300y = 6000$

$500y - (+300y) = 500y - 300y = 200y$

$7600 - 6000 = 1600$

$$\frac{200y}{200} = \frac{1600}{200}$$

$$y = 8$$

STEPS:

1. Choose a variable to eliminate

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3. **Add** or **subtract** the equations to eliminate the variable

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(1) $300j + 500y = 7600$

300 x (2) $j + y = 20$

(1) $300j + 500y = 7600$

(2) **–** $300j + 300y = 6000$

$$\frac{200y}{200} = \frac{1600}{200}$$

$$y = 8$$

Sub **y = 8** into **(1)** or **(2)** →easier to sub into **(2)**

$$j + y = 20$$

$$j + 8 = 20$$

$$j = 20 - 8$$

$$j = 12$$

STEPS:

1. Choose a variable to eliminate

→ Choose a variable where <u>both</u> co-efficients are the same

2. If necessary, **multiply** the equations by a factor to make the co-efficients the same

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(1) $300j + 500y = 7600$

(2) **–** $300j + 300y = 6000$

$$\frac{200y}{200} = \frac{1600}{200}$$

$$y = 8$$

$j + y = 20$

$j + 8 = 20$

$j = 20 - 8$

$j = 12$

Arnold bought **8 Yeezys** and **12 Jordans**

STEPS:

1. Choose a variable to eliminate

→ Choose a variable where <u>both</u> co-efficients are the same

2. If necessary, **multiply** the equations by a factor to make the co-efficients the same

3. **Add** or **subtract** the equations to eliminate the variable

4. Solve for the remaining variable

5. Substitute this variable into **(1)** or **(2)** to solve for the other

6. State the solution