ST. JEAN DE BREBEUF MATHEMATICS

CHAPTER 7.6

SOLVING PROBLEMS INVOLVING EXPONENTIAL GROWTH and DECAY

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KEY CONCEPTS

Doubling time is the time it takes for a population to **double in size**.

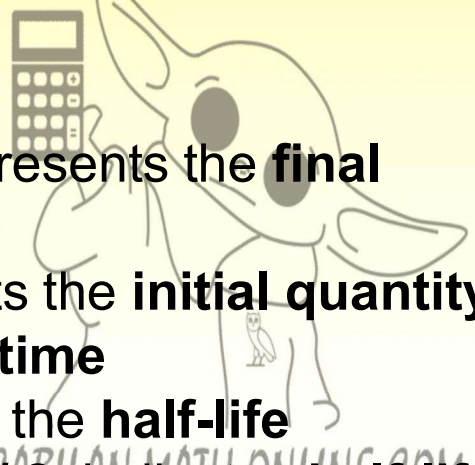
The relation for doubling is $P = P_0(2)^{\frac{t}{d}}$

where ***P*** represents the **population**
P_0 represents the **initial population**,
t represents **time**,
d represents the **doubling time**.
The base, **2**, indicates **doubling**.

Half-life is the time it takes for a quantity to decay to half its original amount.

The relation for half-life is $M = M_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

where ***M*** represents the **final quantity**
M_0 represents the **initial quantity**
t represents **time**
h represents the **half-life**
The base, **1 / 2**, indicates **half-life**.



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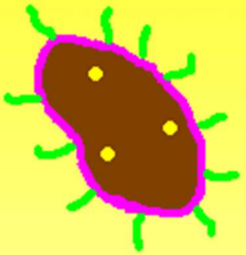
EXAMPLE 1 *Exponential Growth*



A bacterial culture began with **2000** bacteria. Its growth can be modelled using the formula, $N = 2000(2)^{\frac{t}{36}}$, where ***N*** is the **number of bacteria** after ***t*** **hours**.

(a) How many bacteria are present after **36 hours**?

Substitute **t = 36** into the given formula

$$N = 2000(2)^{\frac{t}{36}}$$

$$N = 2000(2)^{\frac{36}{36}}$$

$$N = 2000(2)^{1}$$

$$\boxed{N = 4000}$$

There are **4000 bacteria** present after 36 hours.

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EXAMPLE 1 *Exponential Growth*

A bacterial culture began with **2000** bacteria. Its growth can be modelled using the formula, $N = 2000(2)^{\frac{t}{36}}$, where ***N*** is the **number of bacteria** after ***t*** **hours**.

(b) How many bacteria are present after **72 hours**? How many times has the bacteria doubled after this time?

Substitute **t = 72** into the given formula

$$N = 2000(2)^{\frac{t}{36}}$$

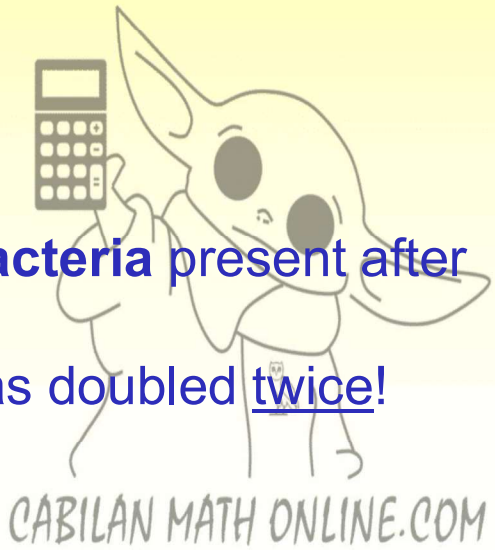
$$N = 2000(2)^{\frac{72}{36}}$$

$$N = 2000(2)^{2}$$

$$\boxed{N = 8000}$$

There are **8000 bacteria** present after 72 hours.

→ The bacteria has doubled twice!



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EXAMPLE 2 *Exponential Decay*

All living organisms contain a known concentration of 1 part per trillion parts of carbon-14. Carbon-14 is a radioactive element. It is used to date ancient artefacts because it has a half-life of about 5730 years after the organism dies.

The formula $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$ is used to calculate the **concentration, *C*, in parts per trillion**, remaining ***n* years** after death.

(a) What would be the concentration of carbon-14 in a piece of cloth (made from plant fibres) after **5730 years**?

Substitute **n = 5730** into given formula

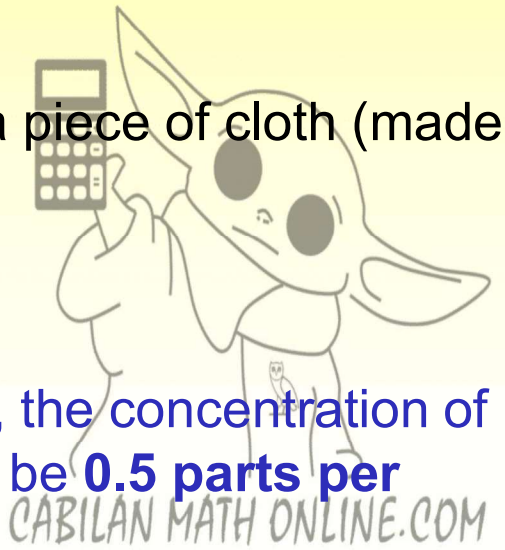
$$C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$$

$$C = \left(\frac{1}{2}\right)^{\frac{5730}{5730}}$$

$$C = \left(\frac{1}{2}\right)^{1}$$

$$\boxed{C = 0.5}$$

After 5730 years, the concentration of carbon-14 would be **0.5 parts per trillion**



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(b) What would be the concentration of carbon-14 in a mummified corpse after **22920 years**? How many *half-lives* is this?

Substitute n **= 22920** into given formula

$$C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$$

$$C = \left(\frac{1}{2}\right)^{\frac{22920}{5730}}$$

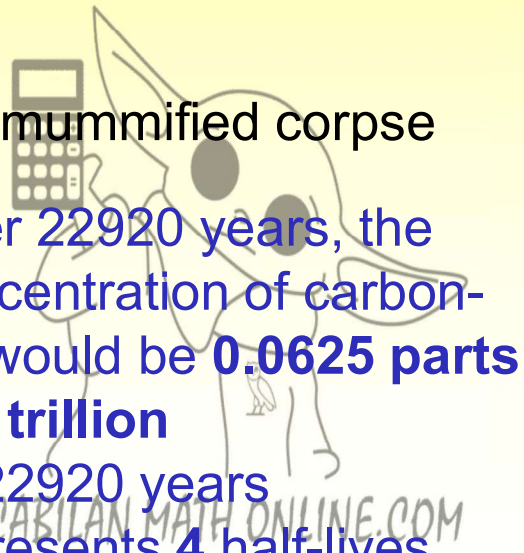
$$C = \left(\frac{1}{2}\right)^{4}$$

$$C = \frac{1}{16}$$

$$C = 0.0625$$

After 22920 years, the concentration of carbon-14 would be **0.0625 parts per trillion**

→ 22920 years represents **4** half-lives



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Homework:

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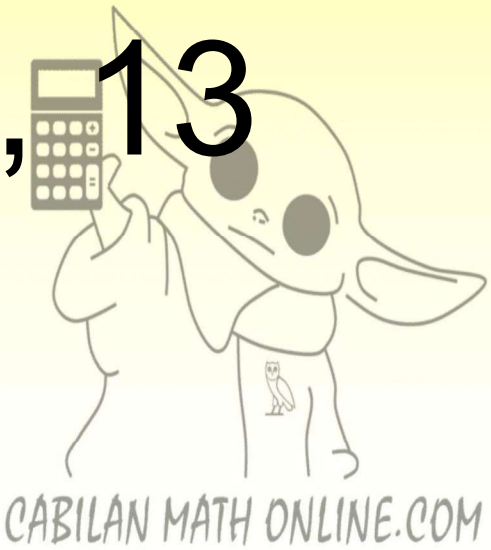
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