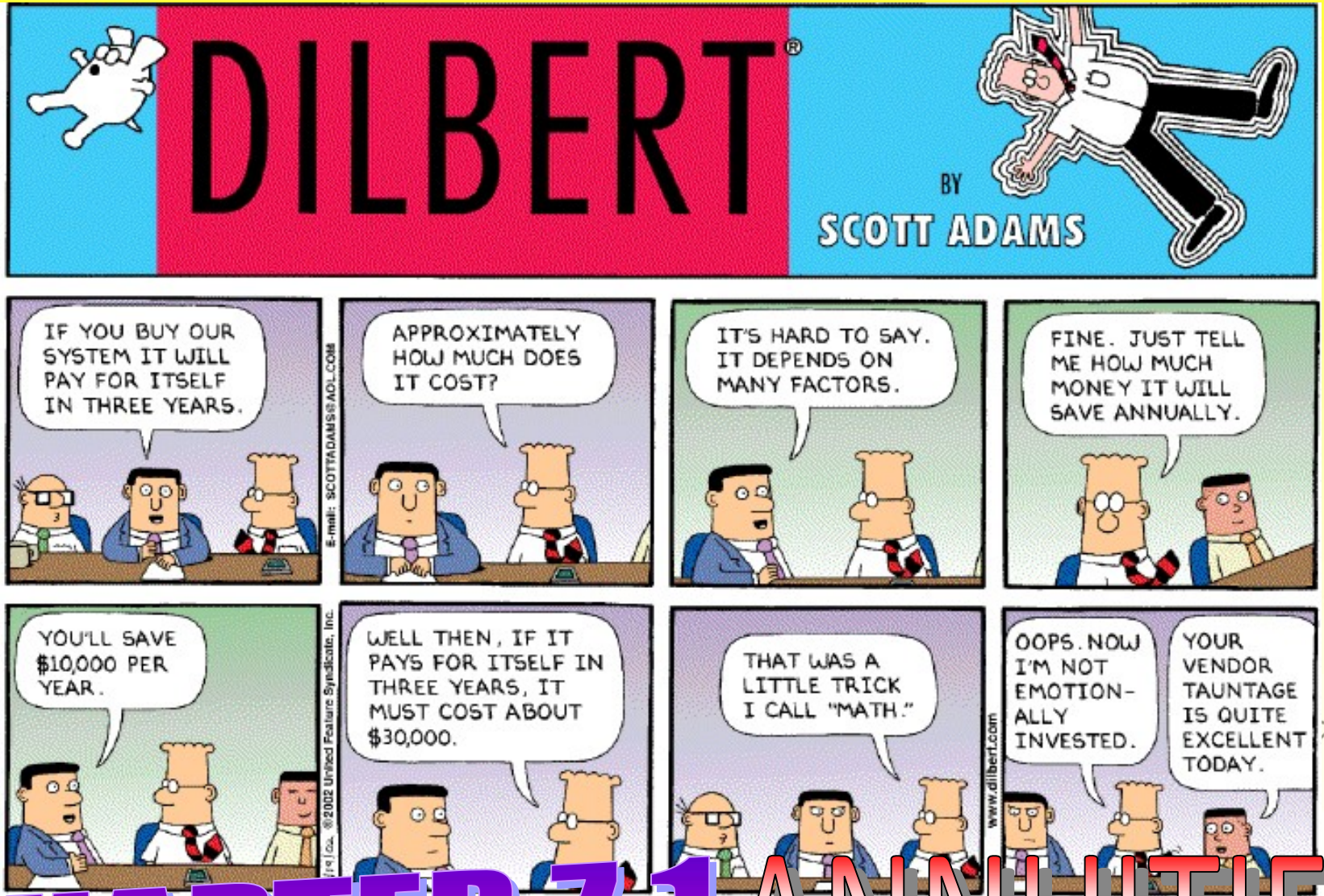
ST. JEAN DE BREBEUF MATHEMATICS



CHAPTER 7.1 ANNUITIES

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REVIEW TERMINOLOGY

1 year (annually) = **365** days
= **12** months
= **52** weeks
= **26** bi-weeks (every other week)

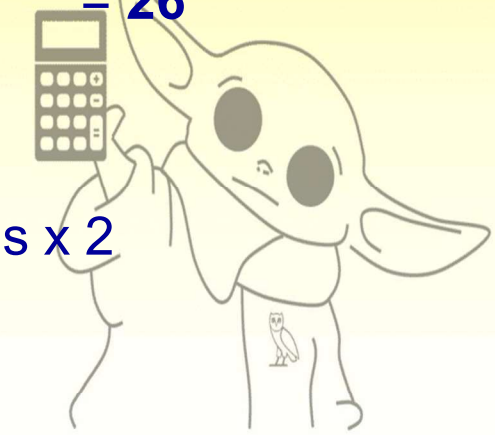
Semi-annually = **2x** per year or every **6** months

Quarterly = **4x** per year or every **3** months

Semi-monthly = **2x** per month = **24** per year

52 weeks / 2
= **26**

12 months x 2
= **24**



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CHAPTER 7.1 ANNUITIES

KEY CONCEPTS

An **annuity** is a series of equal deposits (or *payments*) made at equal intervals of time

→ An **ordinary annuity** is paid at the end of the interval (example: loan payment)

→ An **annuity due** is payable at the beginning of the interval (example: a car payment)

FUTURE VALUE

The **future value** of an annuity is calculated as the sum of a series of *compound interest* calculations

FORMULA (**FV = FUTURE VALUE**)

$$FV = \frac{PMT[(1+i)^n - 1]}{i}$$

When solving for the **payment amount**, the formula can be *re-arranged to*

$$PMT = \frac{iFV}{(1+i)^n - 1}$$

PRESENT VALUE

The **present value** of an annuity is an invested amount that generates a series of future payments, or a loaned amount that requires a series of future payments

FORMULA (**PV = PRESENT VALUE**)

$$PV = \frac{PMT[1-(1+i)^{-n}]}{i}$$

When solving for the **payment amount**, the formula can be *re-arranged to*

$$PMT = \frac{iPV}{1-(1+i)^{-n}}$$

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CHAPTER 7.1 ANNUITIES

FUTURE VALUE

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FORMULA (**FV = FUTURE VALUE**)

$$FV = \frac{PMT[(1+i)^n - 1]}{i}$$

When solving for the **payment amount**, the formula can be *re-arranged to*

$$PMT = \frac{iFV}{(1+i)^n - 1}$$

For both formulas,

PMT = Regular payment amount or amount of investment

PRESENT VALUE

The **present value** of an annuity is an invested amount that generates a series of future payments, or a loaned amount that requires a series of future payments
FORMULA (**PV = PRESENT VALUE**)

$$PV = \frac{PMT[1-(1+i)^{-n}]}{i}$$

When solving for the **payment amount**, the formula can be *re-arranged to*

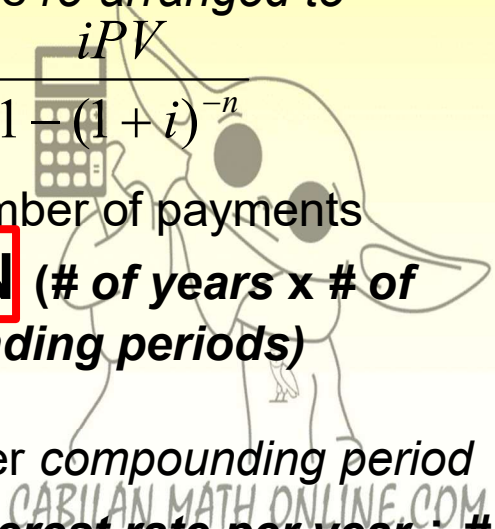
$$PMT = \frac{iPV}{1-(1+i)^{-n}}$$

n = Total number of payments

where **n = yN** ***(# of years x # of compounding periods)***

i = Interest rate per *compounding period*

where **i = r / N** ***(interest rate per year ÷ # of compounding periods)***



CHAPTER 7.1 ANNUITIES

The **TVM Solver** on a graphing calculator can also be used to calculate *future value, present value* and *payment*.

To enter the **TVM Solver**, press **APPS → 1:Finance → 1:TVM Solver**

N = Total # of payments (*years* x *# of payments per year*)

I% = Interest rate *per year*

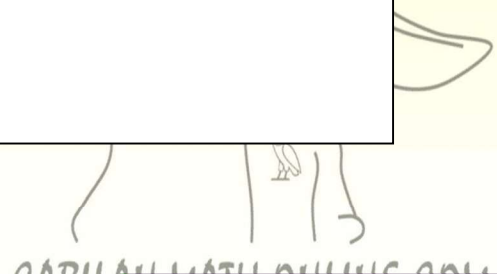
PV = Present value

PMT = Payment/investment amount (entered as a *negative*)

FV = Future value

P/Y = # of payments per year

C/Y = # of compounding periods per year



TVM SOLVER

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \qquad PMT = \frac{iFV}{(1+i)^n - 1} \qquad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \qquad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

EXAMPLE 1 *The Future Value of an Ordinary Simple Annuity*

Emilia recent started a part-time job. She is saving for a car which she will need when she attends college in **2 years**. She plans to deposit **$400** at the end of each month into an account that pays **3.6% per year**, compounded *monthly*.

3.6 / 100
= 0.036

12

(a) How much will Emilia have in **2 years**?

PMT
= 400

i = r / N
= 0.036 / 12
= 0.003

n = yN
= 2(12)
= 24

$$FV = \frac{PMT[(1+i)^n - 1]}{i}$$

$$FV = \frac{400[(1+0.003)^{24} - 1]}{0.003}$$

$$FV = \frac{400[(1.003)^{24} - 1]}{0.003}$$

$$FV = \frac{400[1.0745 - 1]}{0.003}$$

$$FV = \frac{400[0.0745]}{0.003}$$

$$FV = \frac{29.8158}{0.003}$$

$$FV = 9938.60$$

Emilia will have **$9938.60** in 2 years.

TVM SOLVER

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \qquad PMT = \frac{iFV}{(1+i)^n - 1} \qquad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \qquad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

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Emilia recent started a part-time job. She is saving for a car which she will need when she attends college in **2 years**. She plans to deposit **$400** at the end of each month into an account that pays **3.6% per year**, compounded *monthly*.

0.036 **12**

(b) How much will Emilia have in **6 months**? 6 months = **0.5** years

PMT
= **400**

i = r / N
= 0.036 / 12
= **0.003**

n = yN
= 0.5(12)
= **6**

$$FV = \frac{PMT[(1+i)^n - 1]}{i}$$

$$FV = \frac{400[(1+0.003)^6 - 1]}{0.003}$$

$$FV = \frac{400[(1.003)^6 - 1]}{0.003}$$

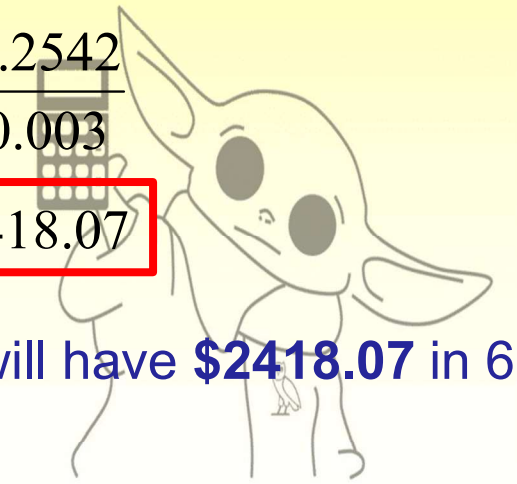
$$FV = \frac{400[1.0181 - 1]}{0.003}$$

$$FV = \frac{400[0.0181]}{0.003}$$

$$FV = \frac{7.2542}{0.003}$$

$$FV = 2418.07$$

Emilia will have **$2418.07** in 6 months



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EXAMPLE 2 *The Present Value of an Annuity*

DeJuan has just purchased his first car. His bank has given him a car loan with *quarterly* payments of **$1500** for the **first year** of the loan at **12% per year**, compounded *quarterly*.

4

12 / 100
= **0.12**

(a) What is the actual cost of the car if DeJuan pays for it today?

PMT
= **1500**

i = r / N
= 0.12 / 4
= **0.03**

n = yN
= 1(4)
= **4**

$$PV = \frac{PMT[1-(1+i)^{-n}]}{i}$$

$$PV = \frac{1500[1-(1+0.03)^{-4}]}{0.03}$$

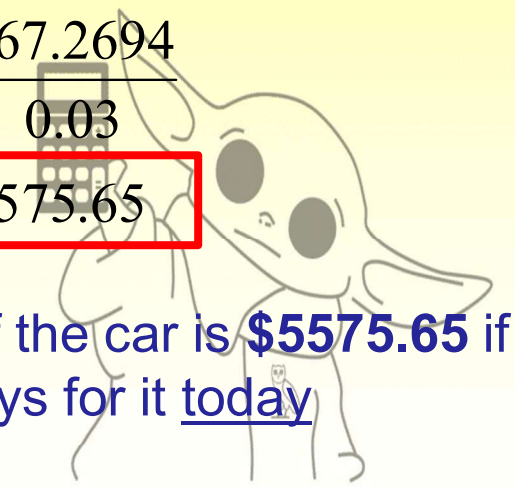
$$PV = \frac{1500[1-(1.03)^{-4}]}{0.03}$$

$$PV = \frac{1500[1-0.8885]}{0.03}$$

$$PV = \frac{1500[0.1115]}{0.03}$$

$$PV = \frac{167.2694}{0.03}$$

$$\boxed{PV = 5575.65}$$



The cost of the car is **$5575.65** if DeJuan pays for it today

TVM SOLVER

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \quad PMT = \frac{iFV}{(1+i)^n - 1} \quad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \quad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

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DeJuan has just purchased his first car. His bank has given him a car loan with *quarterly* payments of **$1500** for the **first year** of the loan at **12% per year**, compounded *quarterly*.

0.12

4

(b) How much **interest** will DeJuan pay by choosing the payment plan?

$$PV = 5575.65$$

Interest (I)

= (n x PMT) – PV

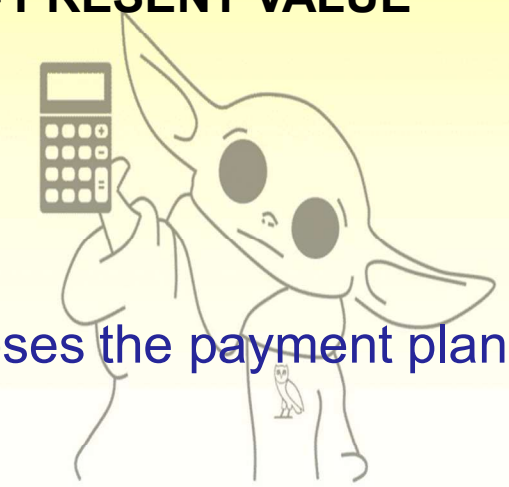
= (# of payments per year x payment amount) – PRESENT VALUE

= (4 x 1500) – 5575.65

= 6000 – 5575.65

= **424.35**

DeJuan will be paying **$424.35** in interest if he chooses the payment plan



TVM SOLVER

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EXAMPLE 3A *The Payment for an Annuity*

Asher recently graduated from college and owes **$16 000** on a student loan that he must begin to repay immediately. Payments are to be made at the end of 6 months, for the next **5 years**. Interest is calculated at **9% per year**, compounded *semi-annually*.

9 / 100
= **0.09**

2

(a) Determine the amount of each semi-annual **payment**

i = r / N
= 0.09 / 2
= **0.045**

PV
= **16 000**

n = yN
= (5)(2)
= **10**

$$PMT = \frac{iPV}{1-(1+i)^{-n}}$$

$$PMT = \frac{(0.045)(16000)}{1-(1+0.045)^{-10}}$$

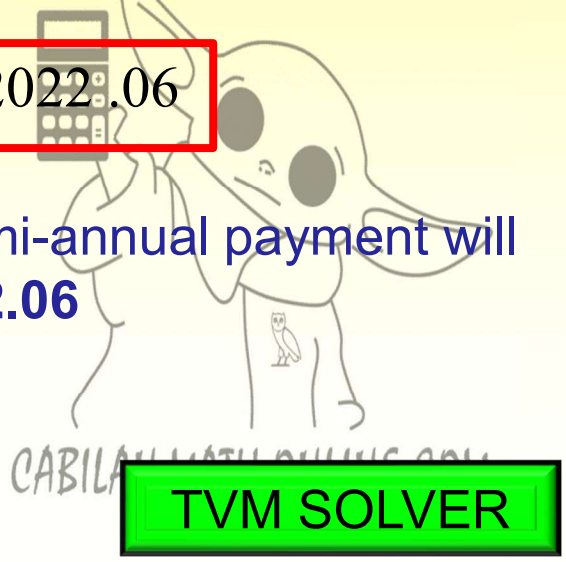
$$PMT = \frac{720}{1-(1.045)^{-10}}$$

$$PMT = \frac{720}{1-0.6439}$$

$$PMT = \frac{720}{0.3561}$$

$$\boxed{PMT = 2022.06}$$

Each semi-annual payment will be **$2022.06**



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i = r / N
= 0.09 / 2
= **0.045**

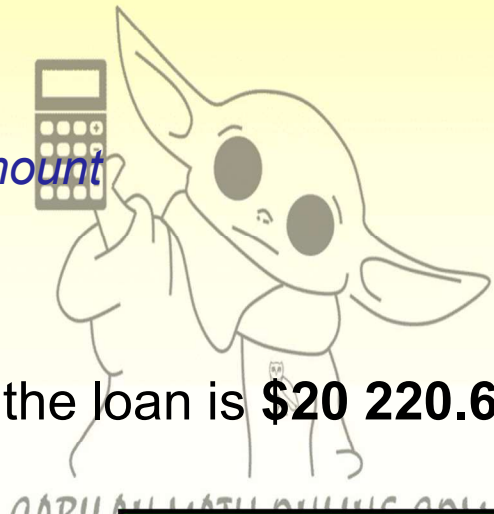
PV
= **16 000**

n = yN
= (5)(2)
= **10**

$$PMT = 2022.06$$

(b) Calculate the **total amount** needed to repay the loan after the **5 years**

Total = n x PMT
= # of payments x payment amount
= 10 x 2022.06
= $20 220.60



The total amount needed to repay the loan is **$20 220.60**

TVM SOLVER

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \qquad PMT = \frac{iFV}{(1+i)^n - 1} \qquad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \qquad \boxed{PMT = \frac{iPV}{1-(1+i)^{-n}}}$$

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Asher recently graduated from college and owes **$16 000** on a student loan that he must begin to repay immediately. Payments are to be made at the end of 6 months, for the next **5 years**. Interest is calculated at **9% per year**, compounded *semi-annually*. **0.09** **2**

i = r / N
= 0.09 / 2
= **0.045**

PV
= **16 000**

n = yN
= (5)(2)
= **10**

$$PMT = 2022.06$$

(c) Calculate the **total amount of *interest*** that Asher will pay
[Use **Interest = (n x PMT) – PV**]

Interest = (n x PMT) – PV
= (10 x 2022.06) – 16 000
= 20220.60 – 16 000
= 4220.60

Asher will be paying **$4220.60** in interest.

TVM SOLVER

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \qquad \boxed{PMT = \frac{iFV}{(1+i)^n - 1}} \qquad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \qquad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

EXAMPLE 3B *The Payment for an Annuity*

Asher's sister, Ashley, is saving her money so that she can purchase a **$2000** computer in **2 years**. Ashley plans to make *quarterly* deposits into a bank account with **1.6%** interest, compounded *quarterly*.

1.6 / 100
= **0.016**

4

(a) Determine the amount of each quarterly **payment/deposit**

i = r / N
= 0.016 / 4
= **0.004**

FV
= **2 000**

n = yN
= (2)(4)
= **8**

$$PMT = \frac{iFV}{(1+i)^n - 1}$$

$$PMT = \frac{(0.004)(2000)}{(1+0.004)^8 - 1}$$

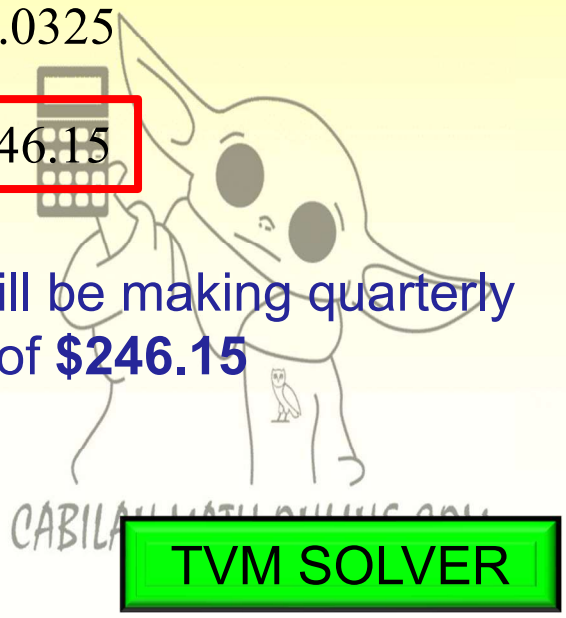
$$PMT = \frac{8}{(1.004)^8 - 1}$$

$$PMT = \frac{8}{1.0325 - 1}$$

$$PMT = \frac{8}{0.0325}$$

$$\boxed{PMT = 246.15}$$

Ashley will be making quarterly deposits of **$246.15**



TVM SOLVER

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \qquad \boxed{PMT = \frac{iFV}{(1+i)^n - 1}} \qquad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \qquad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

EXAMPLE 3B *The Payment for an Annuity*

Asher's sister, Ashley, is saving her money so that she can purchase a **$2000** computer in **2 years**. Ashley plans to make *quarterly* deposits into a bank account with **1.6%** interest, compounded *quarterly*.

0.016 **4**

(b) Calculate the **total amount of *interest*** that Ashley will gain [Use **Interest = FV - (n x PMT)**]

i = r / N
= 0.016 / 4
= **0.004**

$$\boxed{PMT = 246.15}$$

FV
= **2 000**

n = yN
= (2)(4)
= **8**

Interest = FV - (n x PMT)
= 2 000 – (8 x 246.15)
= 2 000 – 1 969.20
= 30.80

Ashley will be earning **$30.80** in interest.

END DAY 1 TVM SOLVER

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \quad PMT = \frac{iFV}{(1+i)^n - 1} \quad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \quad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

EXAMPLE 1 *The Future Value of an Ordinary Simple Annuity*

Emilia recent started a part-time job. She is saving for a car which she will need when she attends college in **2 years**. She plans to deposit **$400** at the end of each month into an account that pays **3.6% per year**, compounded *monthly*.

12

(a) How much will Emilia have in **2 years**.

N = ~~24~~ 12

I% = 3.6

PV = 0

PMT = – 400

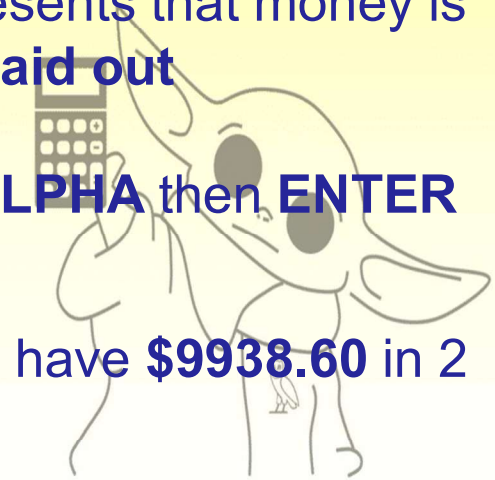
FV = 9938.60

P/Y = 12

C/Y = 12

Entered as a negative
→Represents that money is **being paid out**

Press **ALPHA** then **ENTER**



Emilia will have **$9938.60** in 2 years.

PREV

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \quad PMT = \frac{iFV}{(1+i)^n - 1} \quad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \quad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

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Emilia recent started a part-time job. She is saving for a car which she will need when she attends college in **2 years**. She plans to deposit **$400** at the end of each month into an account that pays **3.6% per year**, compounded *monthly*.

(b) How much will Emilia have in **6 months**?

6 months
= **0.5** years

N =	0.5 x 12
I% =	3.6
PV =	0
PMT =	– 400
FV =	2418.07
P/Y =	12
C/Y =	12

Press **ALPHA** then **ENTER**

Emilia will have **$2418.07** in 6 months

PREV

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \quad PMT = \frac{iFV}{(1+i)^n - 1} \quad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \quad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

EXAMPLE 2 *The Present Value of an Annuity*

DeJuan has just purchased his first car. His bank has given him a car loan with *quarterly* payments of **$1500** for the **first year** of the loan at **12% per year**, compounded *quarterly*.

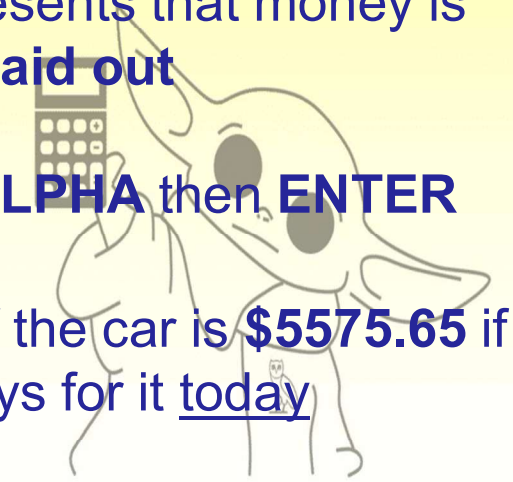
(a) What is the actual cost of the car if DeJuan pays for it today?

N =	1 x 4
I% =	12
PV =	−5575.65
PMT =	– 1500
FV =	0
P/Y =	4
C/Y =	4

Entered as a negative
→Represents that money is **being paid out**

Press **ALPHA** then **ENTER**

The cost of the car is **$5575.65** if DeJuan pays for it today



PREV.

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \quad PMT = \frac{iFV}{(1+i)^n - 1} \quad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \quad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

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DeJuan has just purchased his first car. His bank has given him a car loan with *quarterly* payments of **$1500** for the **first year** of the loan at **12% per year**, compounded *quarterly*.

4

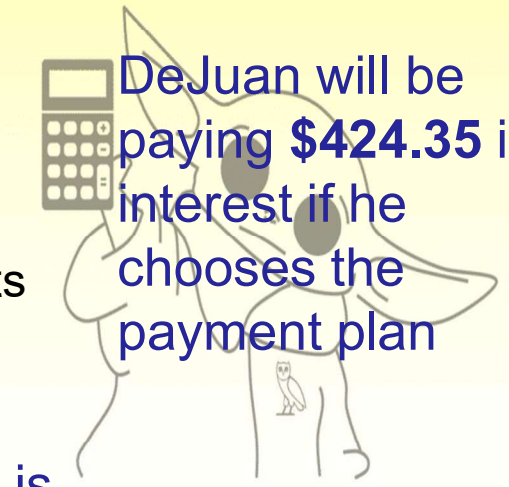
(b) How much **interest** will DeJuan pay by choosing the payment plan?

Using the ∑**Int** function on the Graphing calculator [after solving **(a)**]

Press **2nd** then **MODE**
→ **APPS**
→ **1:Finance**
→ Cursor down to **A:∑Int(** then press **ENTER** once
→Enter **A:∑Int(1,4,2)** then press **ENTER** (**"4"** represents the number of payments in the first year)

– 424.35

Entered as a negative
→ Represents that money is **being paid out**



DeJuan will be paying **$424.35** in interest if he chooses the payment plan

PREV

$$FV = \frac{PMT[(1+i)^n - 1]}{i} \quad PMT = \frac{iFV}{(1+i)^n - 1} \quad PV = \frac{PMT[1-(1+i)^{-n}]}{i} \quad PMT = \frac{iPV}{1-(1+i)^{-n}}$$

EXAMPLE 3A *The Payment for an Annuity*

Asher recently graduated from college and owes **$16 000** on a student loan that he must begin to repay immediately. Payments are to be made at the end of 6 months, for the next **5 years**. Interest is calculated at **9% per year**, compounded *semi-annually*.

2

(a) Determine the amount of each semi-annual **payment**

N = ~~5~~ ~~10~~ 2

I% = 9

PV = 16000

PMT = ~~0~~ 2022.06

FV = 0

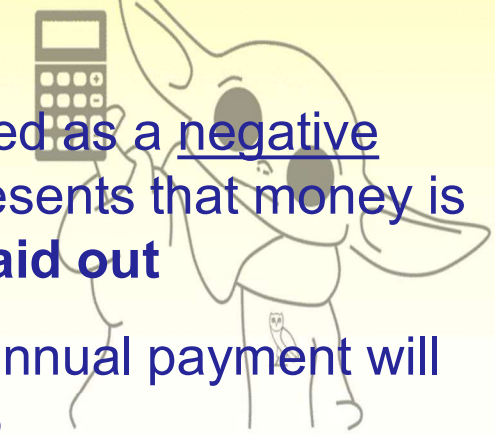
P/Y = 2

C/Y = 2

Entered as **present value**
→ All *loans* must be paid immediately

Expressed as a negative
→ Represents that money is **being paid out**

Each semi-annual payment will be **$2022.06**



PREV

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EXAMPLE 3B *The Payment for an Annuity*

Asher's sister, Ashley, is saving her money so that she can purchase a **$2000** computer in **2 years**. Ashley plans to make *quarterly* deposits into a bank account with **1.6%** interest, compounded *quarterly*.

4

(a) Determine the amount of each quarterly **payment/deposit**

N = ~~4~~8 x 2

I% = 1.6

PV = 0

PMT = ~~0~~ – 246.52

FV = 2000

P/Y = 4

C/Y = 4

Expressed as a negative
→Represents that money is **being paid out**

Press **ALPHA** then **ENTER**

Ashley will be making quarterly deposits of **$246.52**

PREV

HOMEWORK

(Day 1) Page 409 #1, 3, 4, 7, 9, 13

(Day 2) *** Work on Assignment Practice (Using graphing calculator) Page 409 #2, 6, 8, 12, 17