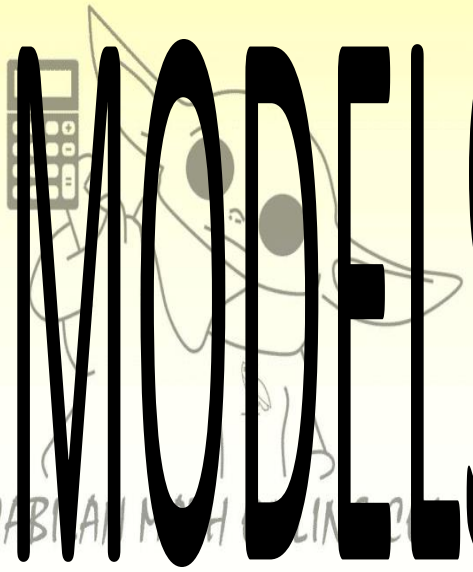
ST. JEAN DE BREBEUF MATHEMATICS

CHAPTER 5.4

ANALYSE GRAPHICAL MODELS



CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

KEY CONCEPTS

The *rate of change* of an **exponential model** depends on its <u>initial conditions</u>

More than one model can sometimes seem to fit a set of data.
→ since different models have different rates of change, the prediction of future behaviour can be an important clue as to which model is more suitable

CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

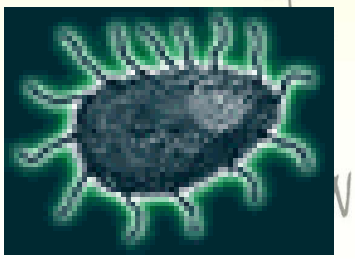
EXAMPLE 1

An important part of medical diagnosis is determining the type of bacteria infecting a patient. Swabs are taken from suspected areas of infection and sent to a microbiology laboratory for testing. The bacteria are transferred to dishes containing nutrient-rich material and are allowed to grow.

Suppose that dish A is infected with **100 bacteria** and dish B is infected with **200 bacteria**. Both types of bacteria grow at a rate of **20% per day**.

(a)Complete the table by showing the daily number of bacteria in each dish for one week. Calculate the **1st Differences** for both populations

TIME (DAYS)	POPULATION A	1st DIFF.
0	100	
1	100 x 1.20 = **120**	120 – 100= **+20**
2	120 x 1.20 = **144**	144 – 120= **+24**
3	144 x 1.20 = **173**	173 – 144= **+29**
4	173 x 1.20 = **208**	**+35**
5	208 x 1.20 = **250**	**+42**
6	250 x 1.20 = **300**	**+50**
7	300 x 1.20 = **360**	**+60**



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CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

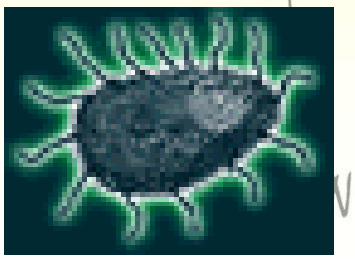
EXAMPLE 1

An important part of medical diagnosis is determining the type of bacteria infecting a patient. Swabs are taken from suspected areas of infection and sent to a microbiology laboratory for testing. The bacteria are transferred to dishes containing nutrient-rich material and are allowed to grow.

Suppose that dish A is infected with **100 bacteria** and dish B is infected with **200 bacteria**. Both types of bacteria grow at a rate of **20% per day**.

(a)Complete the table by showing the daily number of bacteria in each dish for one week. Calculate the **1st Differences** for both populations

TIME (DAYS)	POPULATION B	1st DIFF.
0	200	
1	200 x 1.20 = **240**	240 – 200= **+40**
2	240 x 1.20 = **288**	288 – 240= **+48**
3	288 x 1.20 = **346**	346 – 288= **+58**
4	346 x 1.20 = **415**	**+69**
5	415 x1.20 = **498**	**+83**
6	498 x 1.20 = **598**	**+100**
7	598 x 1.20 = **718**	**+120**



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CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

EXAMPLE 1

(b) Compare the *rates of change* for the two populations. What are the suitable *units* for the rates of change?

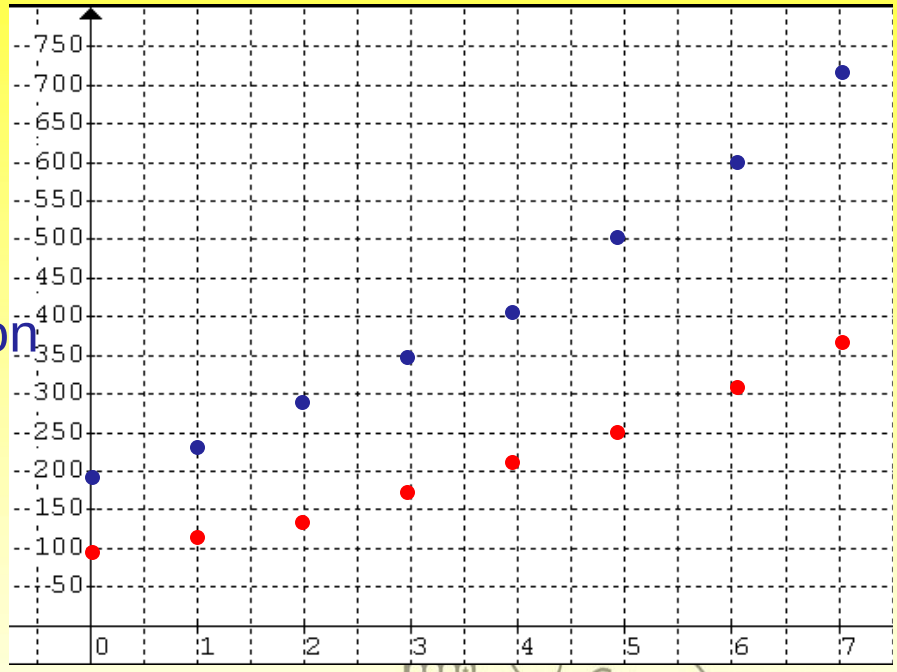
$$Rate\ of\ change = \frac{Population}{Time}$$

$$Rate\ of\ change = \frac{Bacteria}{Day}$$

The units for rate of change is ***bacteria per day****.*

(c) Draw a graph for each population(1 2) on the same set of axes, with **Time** on the *horizontal* axis and **Population** on the *vertical* axis

# of Bacteria vs. Days



- Population A
- Population B

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CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

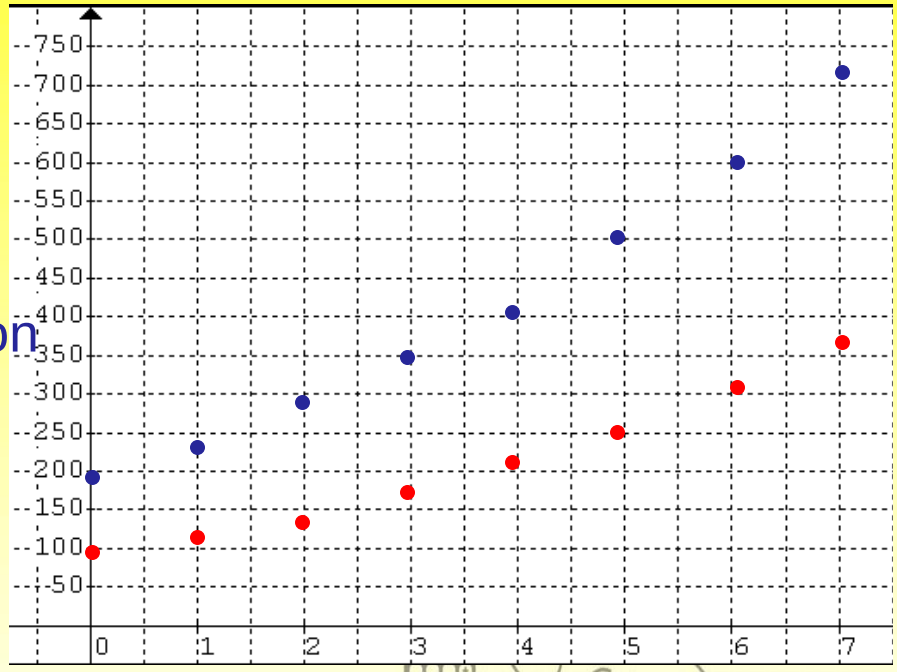
EXAMPLE 1

(d) Are the two populations *increasing* or *decreasing*?

INCREASING

* As time increases, population increases

# of Bacteria vs. Days



- Population A
- Population B

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CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

EXAMPLE 2

Leaky Compressor

Dwight owns a construction business. He uses a compressor attached to a storage tank to maintain a supply of compressed air for power tools. He suspects a leak either in the tank itself or in the valve assemblies between the compressor and the tank.

If the leak is in the tank, the pressure should drop according to an *exponential* model. If the leak is in the valve, the pressure should drop according to a *linear* model.

Dwight measured the pressure in pounds per square inch (psi) every hour for 5 hours.

TIME (h)	PRESSURE (PSI)	1st DIFFERENCES
0	3000	
1	2950	
2	2900	
3	2850	
4	2800	
5	2750	

TIME (h)	PRESSURE (PSI)	RATIOS
0	3000	
1	2950	
2	2900	
3	2850	
4	2800	
5	2750	

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CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

EXAMPLE 2

Leaky Compressor

(a) Calculate the **1st Differences** and the **Ratios**.

(i) Explain why a **linear model** would be appropriate for this situation

***1st differences** are constant*

(ii) Explain why an **exponential model** would also be appropriate for this situation

***Ratios** are constant*

TIME (h)	PRESSURE (PSI)	1st DIFFERENCES
0	3000	
		2950 – 3000 = **– 50**
1	2950	
		2900 – 2950 = **– 50**
2	2900	
		2850 – 2900 = **– 50**
3	2850	
		– 50
4	2800	
		– 50
5	2750	

TIME (h)	PRESSURE (PSI)	RATIOS
0	3000	
		2950 ÷ 3000 = **+0.98**
1	2950	
		2900 ÷ 2950 = **+0.98**
2	2900	
		2850 ÷ 2900 = **+0.98**
3	2850	
		+0.98
4	2800	
		+0.98
5	2750	

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CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

EXAMPLE 2

Leaky Compressor

(b) Using technology (graphing calculator), perform **linear** and **exponential regression** to determine equations for both models

TIME (h)	PRESSURE (PSI)	1st DIFFERENCES
0	3000	
		– 50
1	2950	
		– 50
2	2900	
		– 50
3	2850	
		– 50
4	2800	
		– 50
5	2750	

LINEAR (store as *Y1*)

LinReg

$y = ax+b$

$a = -50$

$b = 3000$

$r^2 = 1$

$r = -1$

EQUATION:

$y = -50x + 3000$

EXPONENTIAL (store as *Y2*)

ExpReg

$y = a*b\text{^}x$

$a = 3001.42$

$b = 0.983$

$r^2 = 0.999$

$r = -0.999$

EQUATION:

$y = 3001.42(0.983)^x$

TIME (h)	PRESSURE (PSI)	RATIOS
0	3000	
		+0.98
1	2950	
		+0.98
2	2900	
		+0.98
3	2850	
		+0.98
4	2800	
		+0.98
5	2750	

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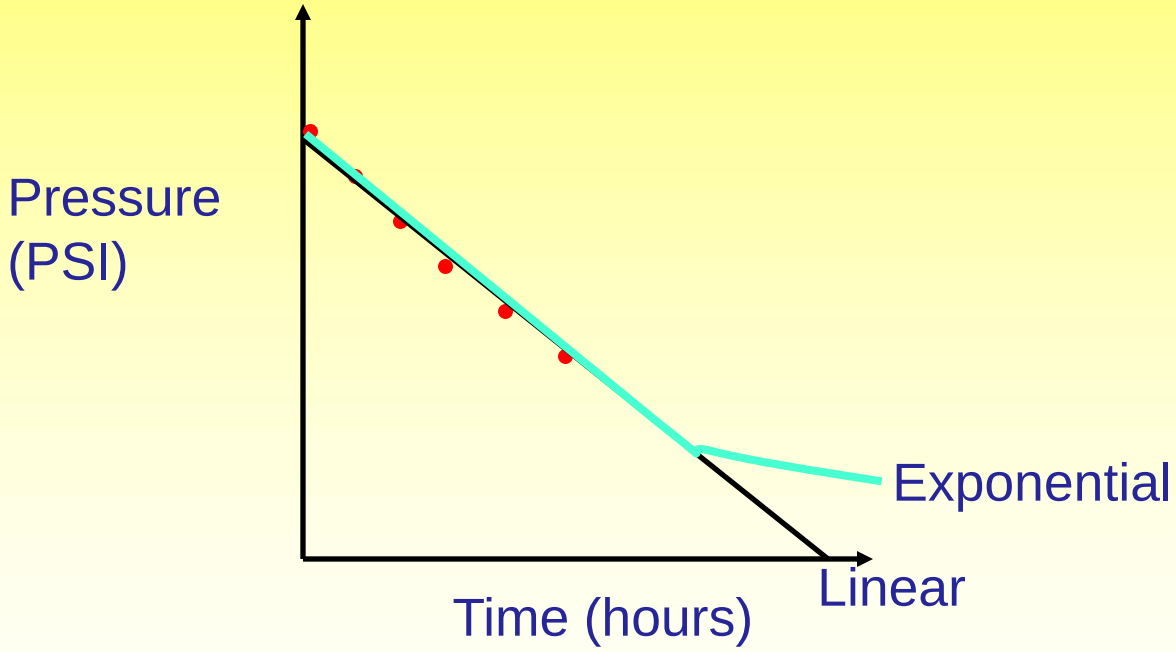
CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

EXAMPLE 2

Leaky Compressor

(c) Press **GRAPH** and sketch both graphs below on the same axes



TIME (h)	PRESSURE (PSI)	1st DIFFERENCES
0	3000	
1	2950	− 50
2	2900	− 50
3	2850	− 50
4	2800	− 50
5	2750	− 50

TIME (h)	PRESSURE (PSI)	RATIOS
0	3000	
1	2950	+0.98
2	2900	+0.98
3	2850	+0.98
4	2800	+0.98
5	2750	+0.98

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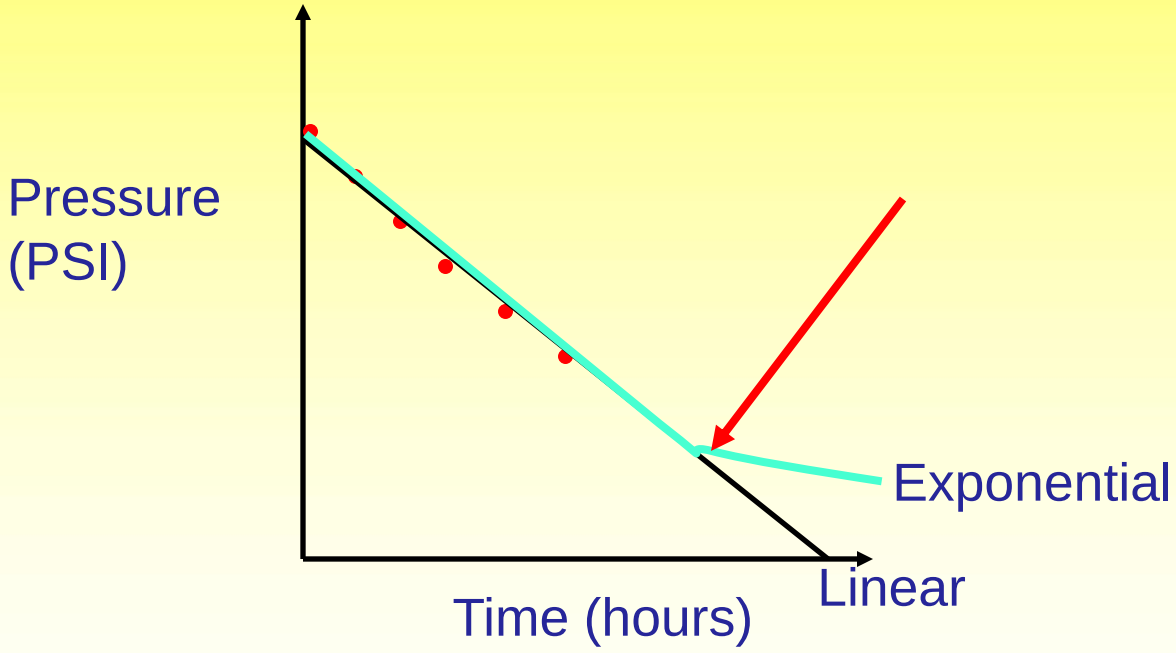
CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

EXAMPLE 2

Leaky Compressor

(c) Press **GRAPH** and sketch both graphs below on the same axes



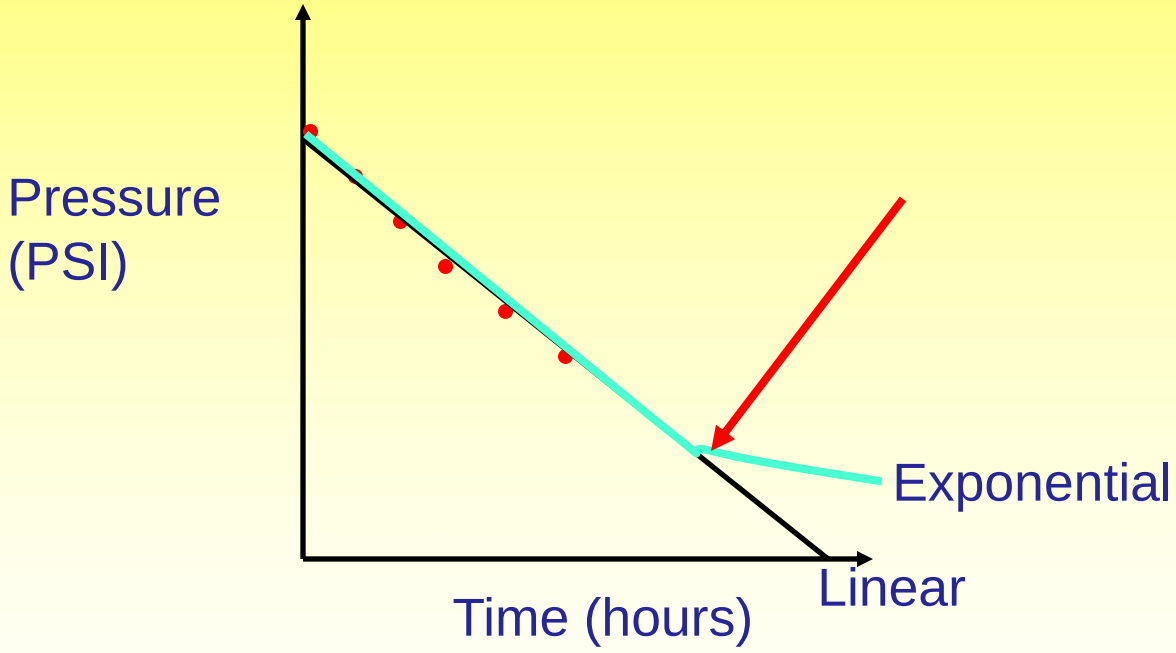
(d) After how many hours does it take before the graphs show a *visible* difference? Press **TRACE** and use the Cursor keys.

*Approximately after **10 – 15 hours***

CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

EXAMPLE 2

Leaky Compressor



(e) Dwight left the tank overnight. The next morning, after **14 h**, the pressure was **2300 psi**. Which model predicts this pressure most accurately? Click on each graph and 14 hours to see which one is closest to 2300 psi.

LINEAR

CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

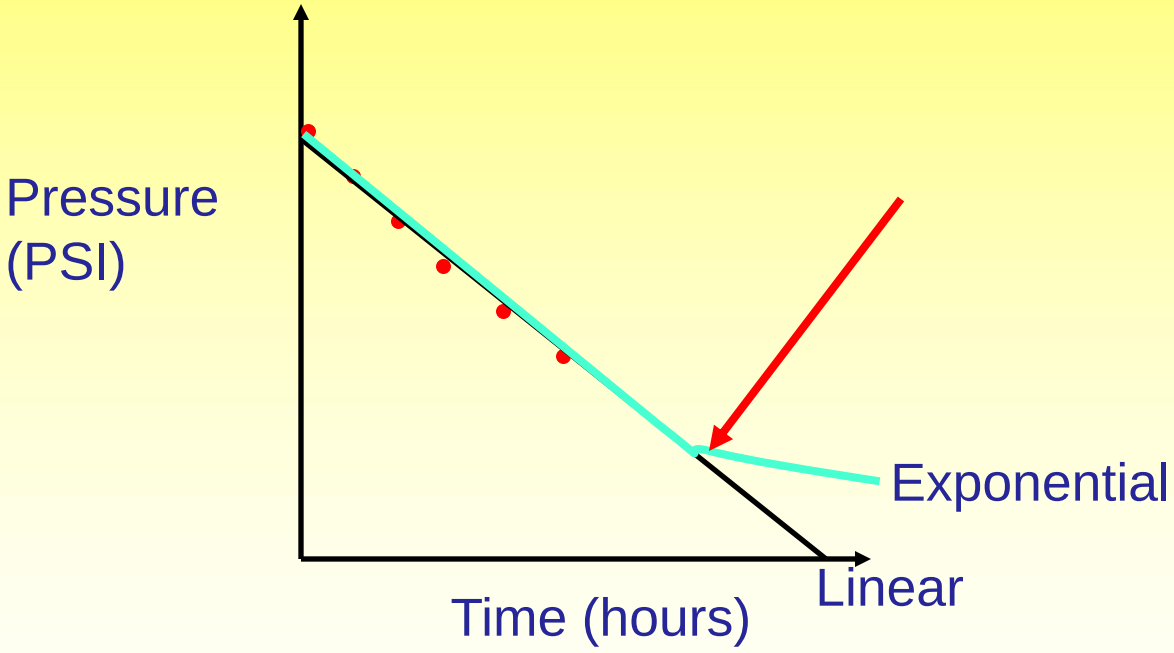
EXAMPLE 2

Leaky Compressor

(f) Which model predicts that the pressure will decrease more slowly?

EXPONENTIAL

→ As time increases, pressure decreases and curve becomes less steep



CHAPTER 5.4 ANALYSE GRAPHICAL MODELS

HOMEWORK / SEATWORK

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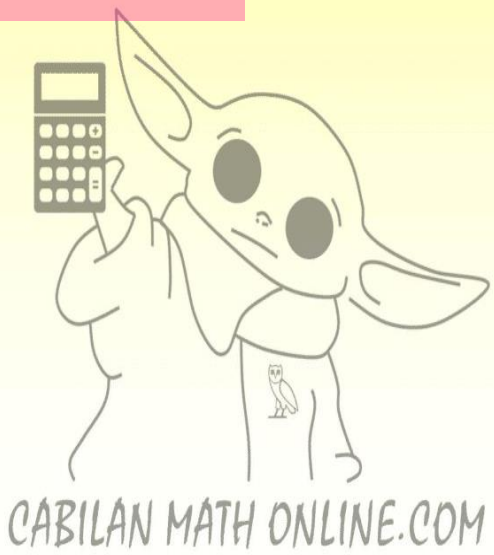
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