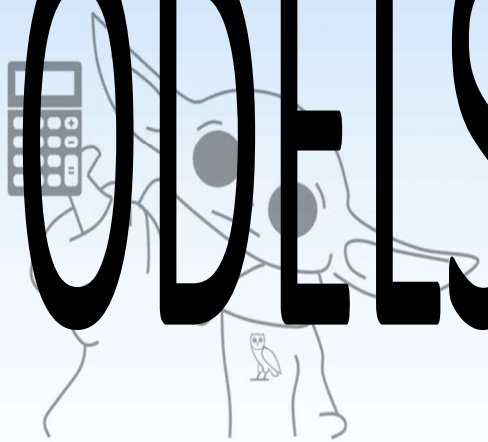
ST. JEAN DE BREBEUF MATHEMATICS

CHAPTER 5.3

EXPONENTIAL MODELS



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CHAPTER 5.3 EXPONENTIAL MODELS

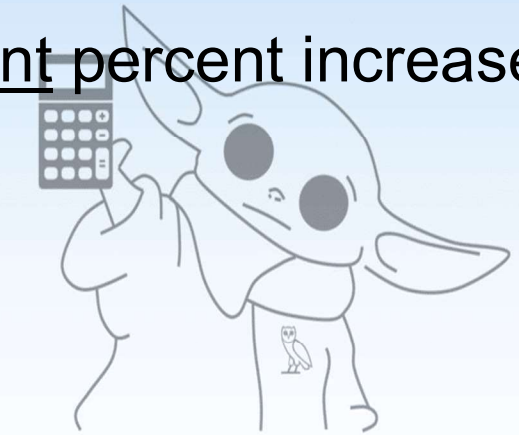
KEY CONCEPTS

For all **exponential** models, the *first and second differences* are always non-constant

The **ratios** can be calculated by dividing successive *y* values from the *previous* y values

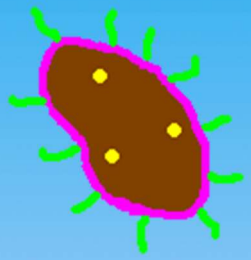
→ When ratios are constant, the relation is **exponential**

In an **exponential relation**, there is a constant percent increase over equal intervals

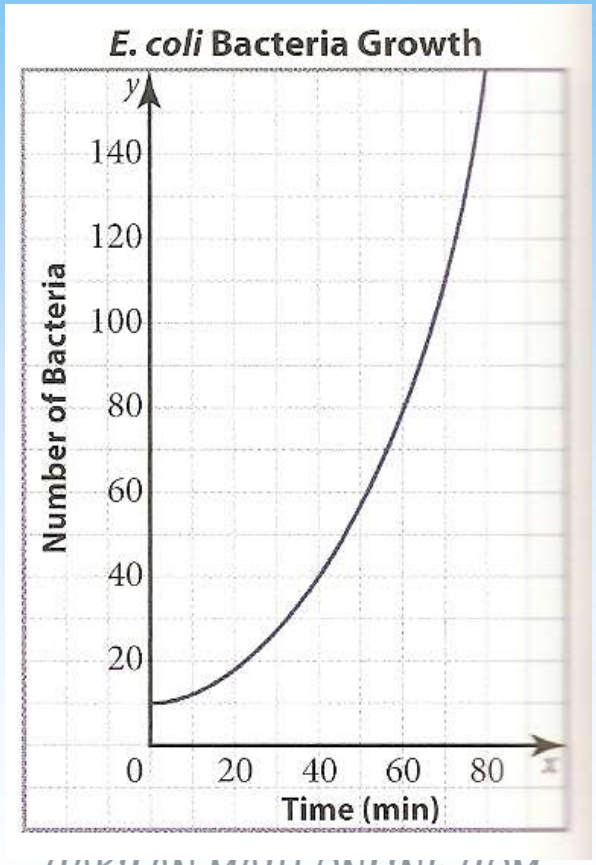


CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 1 *Bacteria Growth*



When some foods, such as fresh meat, are left at room temperature, any bacteria in the food will begin to multiply. If enough bacteria are present, you can become ill from eating the food. This is known as food poisoning. The most common bacteria found in food are salmonella and *E. coli*. The graph shows the growth of an *E. coli* bacteria population over time.



(a) Describe the relationship between number of bacteria and time

As time increases, the number of bacteria increases

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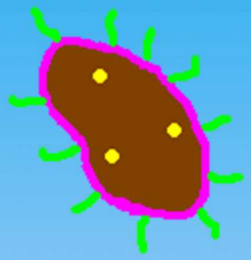
CHAPTER 5.3 EXPONENTIAL MODELS

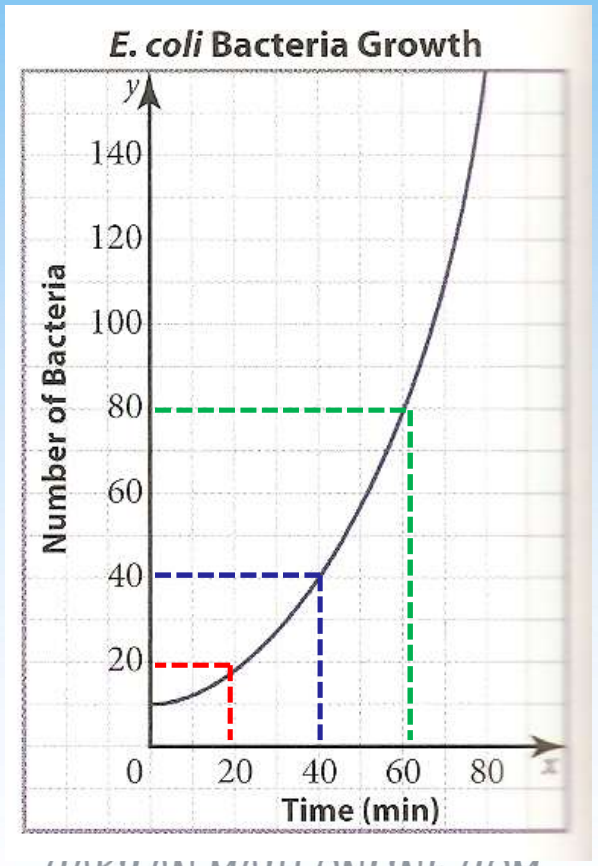
EXAMPLE 1 *Bacteria Growth*



(b) Use the graph to estimate the number of bacteria after each time period

(i) 20 minutes **20** (ii) 40 minutes **40**

(iii) 60 minutes **80**



(c) Calculate the **ratios**

Divide the number of bacteria **after 40 min** by the number **after 20 min**

$$\frac{\#\ of\ Bacteria\ at\ 40\,\text{min}}{\#\ of\ Bacteria\ at\ 20\,\text{min}} = \frac{40}{20} = 2$$

Divide the number of bacteria **after 60 min** by the number **after 40 min**

$$\frac{\#\ of\ Bacteria\ at\ 60\,\text{min}}{\#\ of\ Bacteria\ at\ 40\,\text{min}} = \frac{80}{40} = 2$$

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CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 1 *Bacteria Growth*

Divide the number of bacteria **after 40 min** by the number **after 20 min**

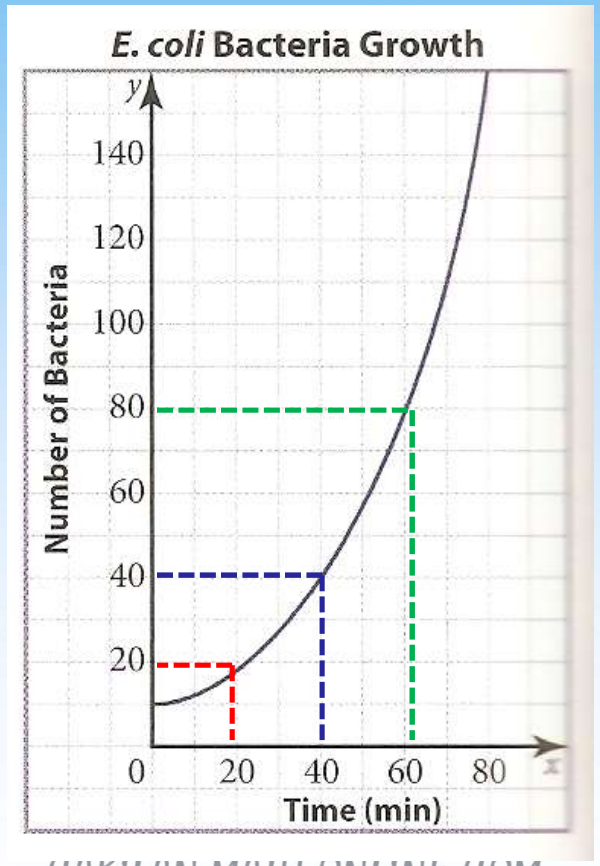
$$\frac{\#\,of\ Bacteria\ at\ 40\,\text{min}}{\#\,of\ Bacteria\ at\ 20\,\text{min}} = \frac{40}{20} = 2$$

Divide the number of bacteria **after 60 min** by the number **after 40 min**

$$\frac{\#\,of\ Bacteria\ at\ 60\,\text{min}}{\#\,of\ Bacteria\ at\ 40\,\text{min}} = \frac{80}{40} = 2$$

(d) What happens to the number of bacteria <u>every 20 minutes</u>?

*The number of bacteria **doubles***



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CHAPTER 5.3 EXPONENTIAL MODELS

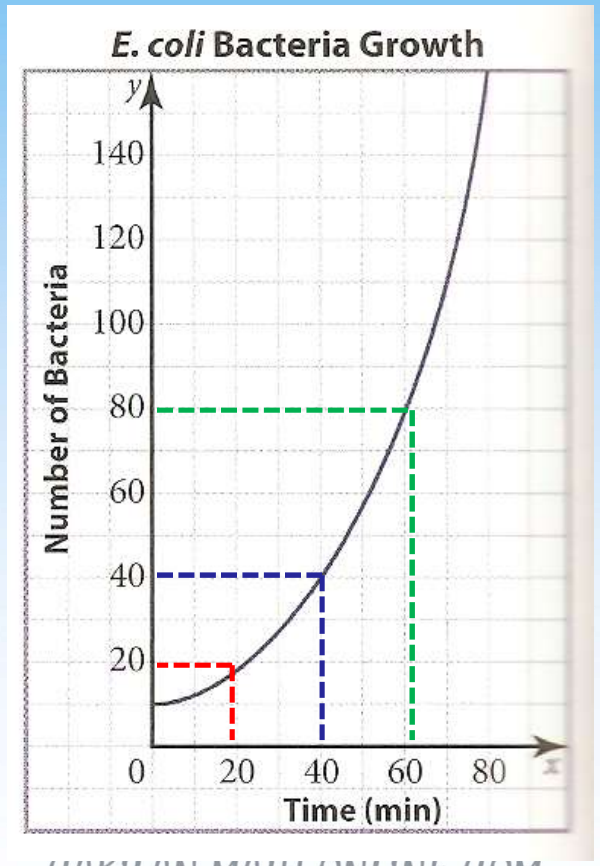
EXAMPLE 1 *Bacteria Growth*

(e) Consider the rate of change of number of bacteria with respect to time. What are suitable **units** for this rate of change?

$$Rate\ of\ change = \frac{\#\ of\ Bacteria}{Time}$$

$$Rate\ of\ change = \frac{Bacteria}{minute}$$

The units for rate of change is **bacteria per minute**



(f) Is the rate of change of number of bacteria with respect to time *increasing, constant* or *decreasing*. Explain.

INCREASING

→ *Curve is going upwards*

→ *Curve is getting steeper*

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CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 2
Smoke Detectors

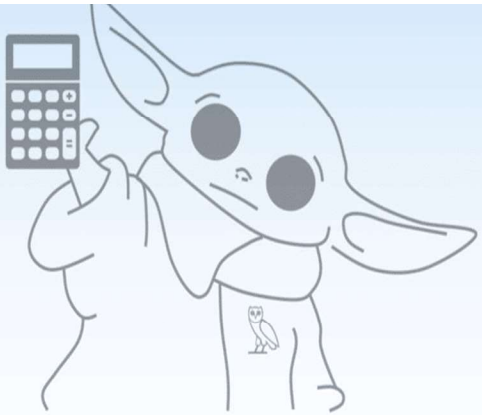
Americium-241 (Am-241) is a manufactured element. It is a silvery radioactive metal, which is used in smoke detectors. Household smoke detectors contain about 200 micrograms (μg) of Am-241. The amount of Am-241 present in the detector *decreases* or *decays* over time. The table shows the mass of Am-241 remaining, in micrograms, over 1000 years.

	YEARS	MASS REMAINING (μg)	1st DIFF.	2nd DIFF.
	0	200		
+ 100	100	170		
+ 100	200	145		
+ 100	300	124		
+ 100	400	105		
+ 100	500	90		
+ 100	600	76		
+ 100	700	65		
+ 100	800	55		
+ 100	900	47		
+ 100	1000	40		

(a) Does the **Years** column show equal time intervals?

Yes

→ *Going up by **100 years***



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CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 2
Smoke Detectors

(b) Calculate the **1st Differences** and **2nd Differences**. Is the relationship *linear, quadratic* or *neither*? Explain.

	YEARS	MASS REMAINING (μg)	1st DIFF.	2nd DIFF.
	0	200		
+ 100	100	170	170 - 200 = – 30	
+ 100	200	145	145 - 170 = – 25	
+ 100	300	124	124 – 145 = – 21	
+ 100	400	105	– 19	
+ 100	500	90	– 15	
+ 100	600	76	– 14	
+ 100	700	65	– 11	
+ 100	800	55	– 10	
+ 100	900	47	– 8	
+ 100	1000	40	– 7	

1st Differences
→Not constant
→Not Linear!

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CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 2
Smoke Detectors

(b) Calculate the **1st Differences** and **2nd Differences**. Is the relationship *linear, quadratic* or *neither*? Explain.

YEARS	MASS REMAINING (μg)	1st DIFF.	2nd DIFF.
0	200		
		– 30	
100	170		+ 5
		– 25	
200	145		+ 4
		– 21	
300	124		+ 2
		– 19	
400	105		+ 4
		– 15	
500	90		+ 1
		– 14	
600	76		+ 3
		– 11	
700	65		+ 1
		– 10	
800	55		+ 2
		– 8	
900	47		+ 1
		– 7	
1000	40		

+ 100
+ 100
+ 100
+ 100
+ 100
+ 100
+ 100
+ 100
+ 100
+ 100

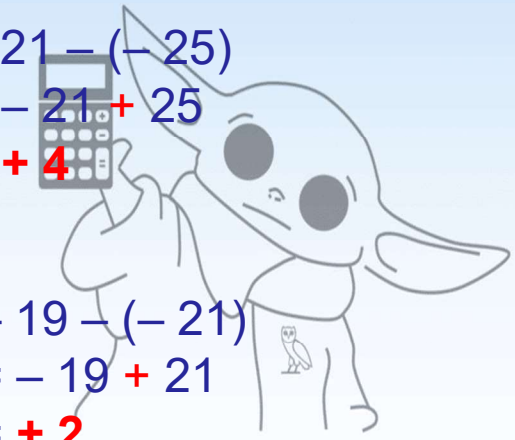
1st Differences
→Not constant
→Not Linear!

2nd Differences
→Not constant
→Not Quadratic!

– 25 – (– 30)
= – 25 + 30
= **+ 5**

– 21 – (– 25)
= – 21 + 25
= **+ 4**

– 19 – (– 21)
= – 19 + 21
= **+ 2**



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CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 2
Smoke Detectors

(c) Calculate the **ratios**. Divide the *second* mass by the first, the *third* mass by the second, etc. How do you know that the relation is **exponential**?

YEARS	MASS REMAINING (μg)	RATIO
0	200	
100	170	+ 0.85
200	145	+ 0.85
300	124	+ 0.85
400	105	+ 0.85
500	90	+ 0.85
600	76	+ 0.85
700	65	+ 0.85
800	55	+ 0.85
900	47	+ 0.85
1000	40	+ 0.85

170 / 200
= **+ 0.85**

145 / 170
= **+ 0.85**

124 / 145
= **+ 0.85**

1st Differences
→Not constant
→Not Linear!

2nd Differences
→Not constant
→Not Quadratic!

Ratios
→Constant!!!
→**EXPONENTIAL!**

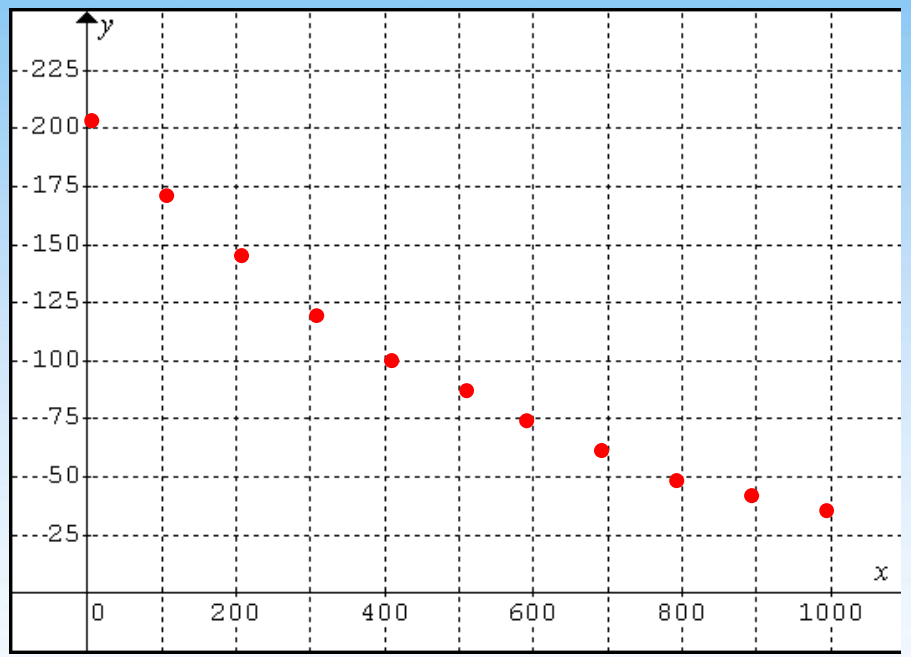
Divide successive *y* values!

CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 2
Smoke Detectors

(d) Draw a graph with **Years** on the *horizontal* axis and **Mass** remaining on the *vertical* axis.

Mass ***(μg)***

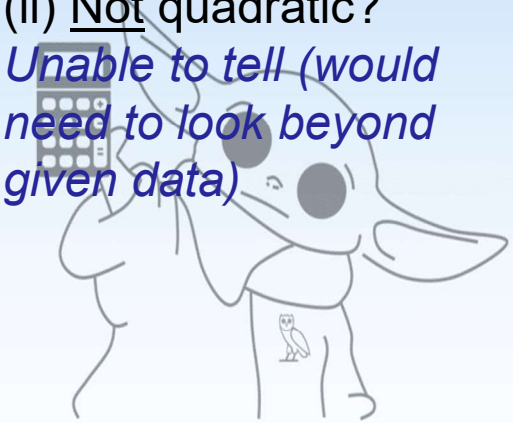


Time (Years)

(e) How do you know that this graph is:

(i) Not linear?
Not a straight line

(ii) Not quadratic?
Unable to tell (would need to look beyond given data)



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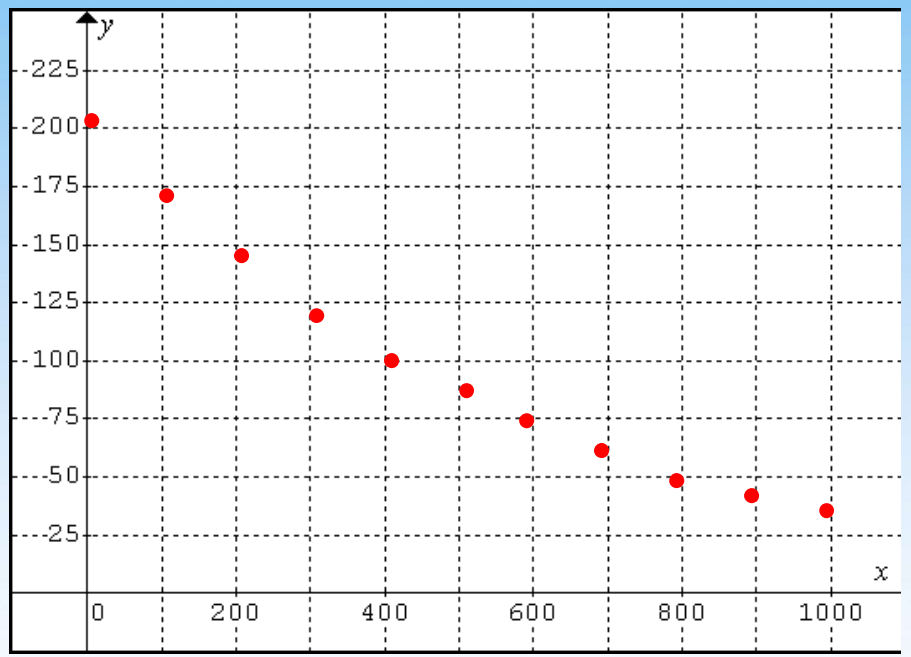
CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 2
Smoke Detectors

(d) Draw a graph with **Years** on the *horizontal* axis and **Mass** remaining on the *vertical* axis.

Mass ***(μg)***



Time (Years)

(f) Compare the table and graph. Does the rate of change of mass remaining with respect to years appear to be *constant, increasing* or *decreasing*? Explain.

Look at **1st Differences**
→They are increasing
BUT...
Graph is decreasing
→ Curve is becoming *less* steep

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CHAPTER 5.3 EXPONENTIAL MODELS

EXAMPLE 2
Smoke Detectors

(g) What are the suitable **units** for the rate of change of mass remaining with respect to years?

$$Rate\ of\ change = \frac{Mass\ remaining}{Time}$$

$$Rate\ of\ change = \frac{\mu g}{Year}$$

The units for the rate of change of mass remaining with respect to years is **micrograms per year**

CHAPTER 5.3 EXPONENTIAL REGRESSION

HOMEWORK / SEATWORK

(Day 1) Page 301 #1 – 6