ST. JEAN DE BREBEUF

MATHEMATICS

CHAPTER 2.5

APPLICATIONS OF TRIGONOMETRY

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TRIGONOMETRY FORMULAS

BASIC TRIGONOMETRIC RATIOS

$$\sin A = \frac{OPPOSITE}{HYPOTENUSE} = \frac{y}{r}$$

$$\cos A = \frac{ADJACENT}{HYPOTENUSE} = \frac{x}{r}$$

$$\tan A = \frac{OPPOSITE}{ADJACENT} = \frac{y}{x}$$

If the problem is modelled by a right triangle, use the **primary trigonometric ratios**.

THE SINE LAW

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

If the problem is modelled by an *acute* or *obtuse* triangle **(non-right angled)**
→with two angles and a given side or
→two sides and an opposite angle, use the **Sine Law**

THE COSINE LAW

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

PYTHAGOREAN THEOREM

$$c^2 = a^2 + b^2$$

$$r = \sqrt{x^2 + y^2} \ (for\ obtuse\ angles)$$

Pythagorean Theorem is *only* used to solve for a missing side in a right angle triangle

CHAPTER 2.5 APPLICATIONS OF TRIGONOMETRY

If the problem is modelled by an *acute* or *obtuse* triangle **(non right angle)**
→with <u>two sides</u> and a <u>contained angle</u> **(SAS)**

→ You use these formulas to solve for **side length**

→ **Three sides (SSS)**, use the **Cosine Law**

→ You use these formulas to solve for **angles**

THE COSINE LAW

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

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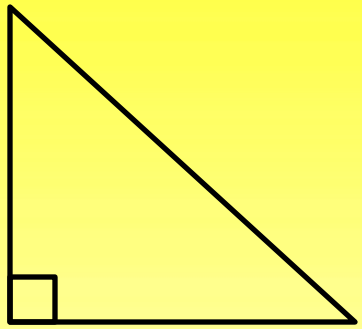
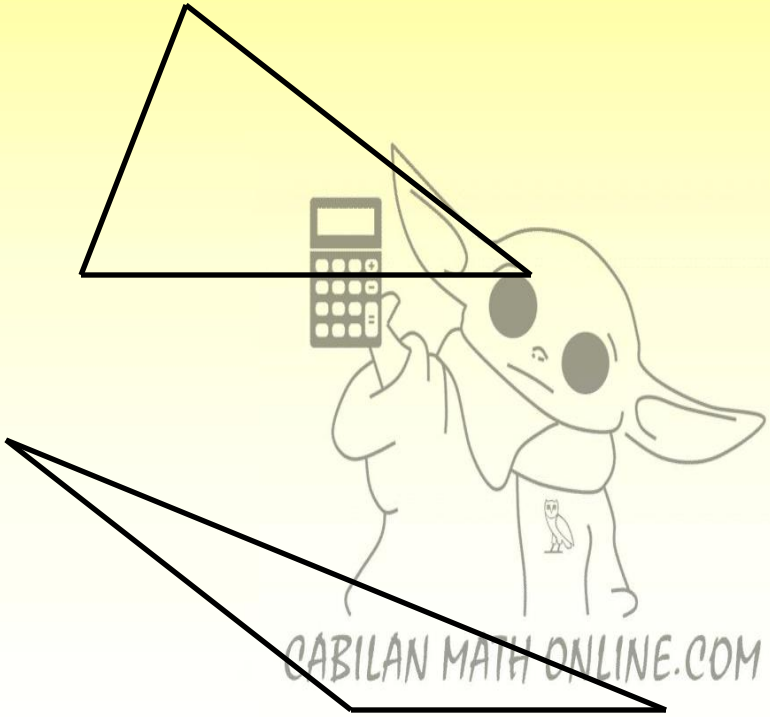
A number of tools and strategies can be used to solve problems involving triangles, including:



The *Primary Trigonometric Ratios* and *Pythagorean Theorem* (used for problems involving *right* triangles)

The Sine Law and The Cosine Law (used for problems involving *acute* or *obtuse* triangles)



Measurement formulas

Geometric properties

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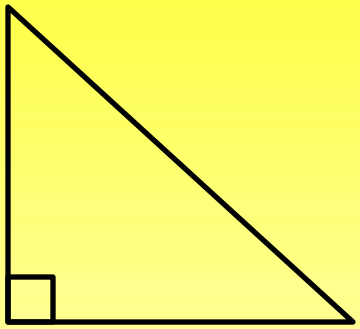
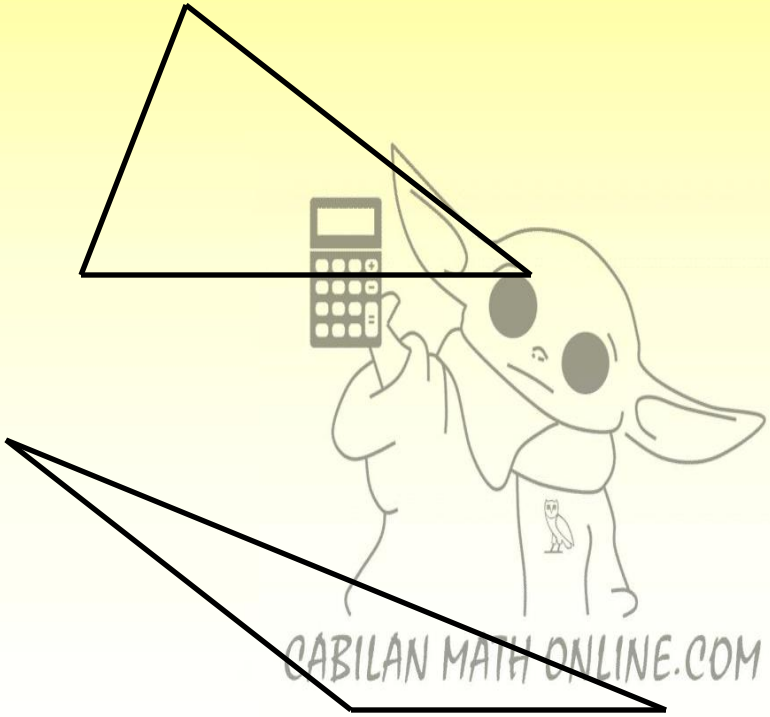
The choice of which tool or strategy to select depends on the *type* of triangle, the information given and in the information required





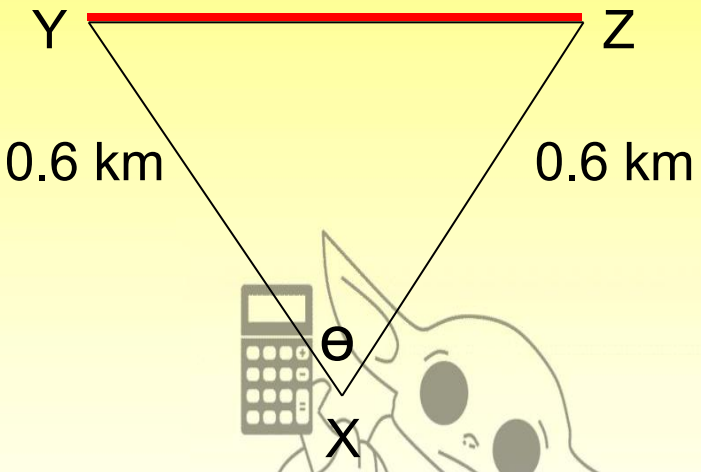
CHAPTER 2.5 APPLICATIONS OF TRIGONOMETRY

EXAMPLE 1 *Set a Swim Course*

The first leg of an Olympic triathlon is a swim. To set the course, which bears the shape of an *isosceles* triangle, two buoys (labelled *Y* and Z) are placed in the water **0.6 km** from the transition zone (as pictured).

To complete this leg of the race, athletes swim in a circuit from the transition zone (X) to the first buoy (Y), then to the second buoy (Z) and back to the transition zone. The total distance of the swim is **1.5 km**.

What is the angle, ɵ, between the lines of sight from the transition zone to the buoys? Express your answer to the nearest degree.



We can use the **Cosine Law**
→We have **SIDE-ANGLE-SIDE (SAS)** (or so we think...) :S
BUT....
We are missing one piece of information

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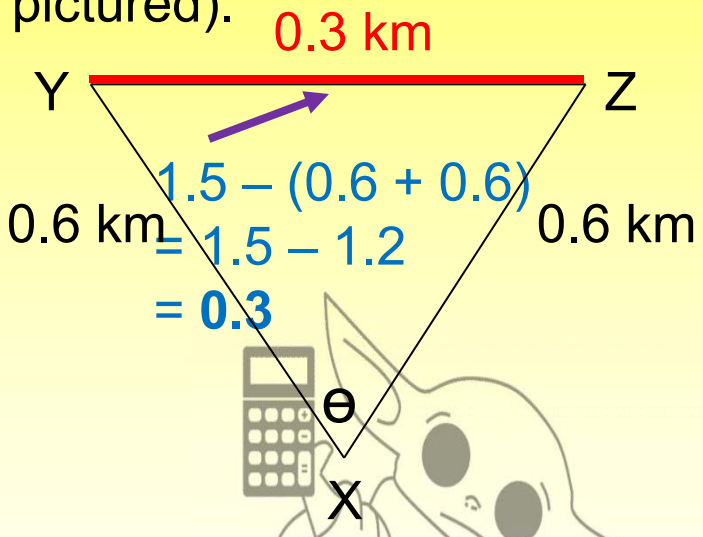

We are missing one piece of information

→Solve for length of third side

→Still use **Cosine Law**

→We now have **SIDE-SIDE-SIDE**

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CHAPTER 2.5 APPLICATIONS OF TRIGONOMETRY

EXAMPLE 1 *Set a Swim Course*

What is the angle, ɵ, between the lines of sight from the transition zone to the buoys? Express your answer to the nearest degree.

$$\cos X = \frac{y^2 + z^2 - x^2}{2yz}$$

$$\cos X = \frac{(0.6)^2 + (0.6)^2 - (0.3)^2}{2(0.6)(0.6)}$$

$$\cos X = \frac{0.36 + 0.36 - 0.09}{0.72}$$

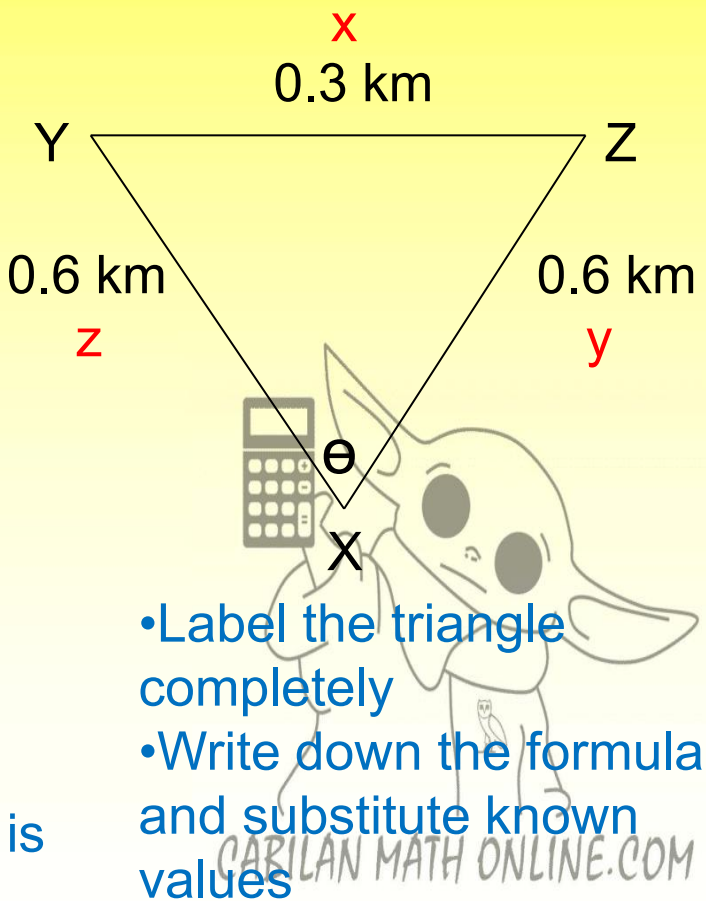
$$\cos X = \frac{0.63}{0.72}$$

$$\cos X = 0.875$$

$$\angle X = \cos^{-1}(0.875)$$

$$\boxed{\angle X = 29°}$$

The angle of sight is **29°**



- Label the triangle completely
- Write down the formula and substitute known values

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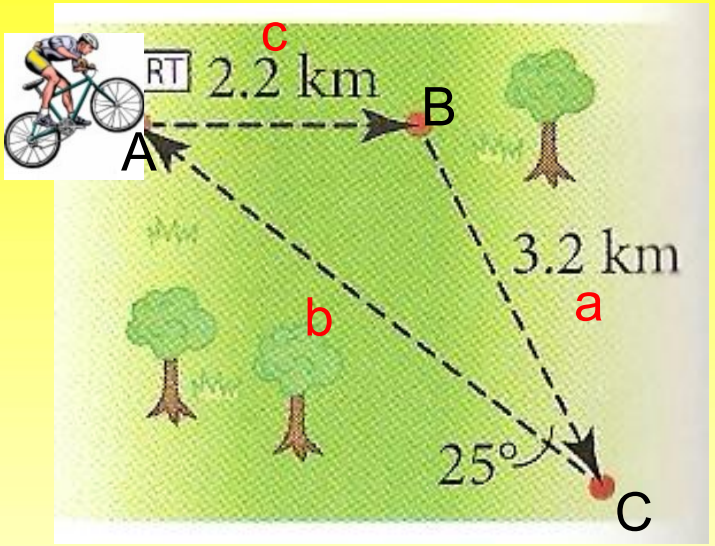
EXAMPLE 2

Set a Bike Course

The second portion of an Olympic triathlon is bicycle ride which has a total distance of **40 km**. The first two parts of the bike route are measured as shown.

After the second leg of the bike route, athletes must make a sharp turn before completing one lap.

(a) What is the distance of the *third* leg of the bike route? Express your answer to the nearest tenth of a km.



We can use the **Sine Law**

→We have an angle and its opposite side

BUT....

We are missing *some* pieces of information

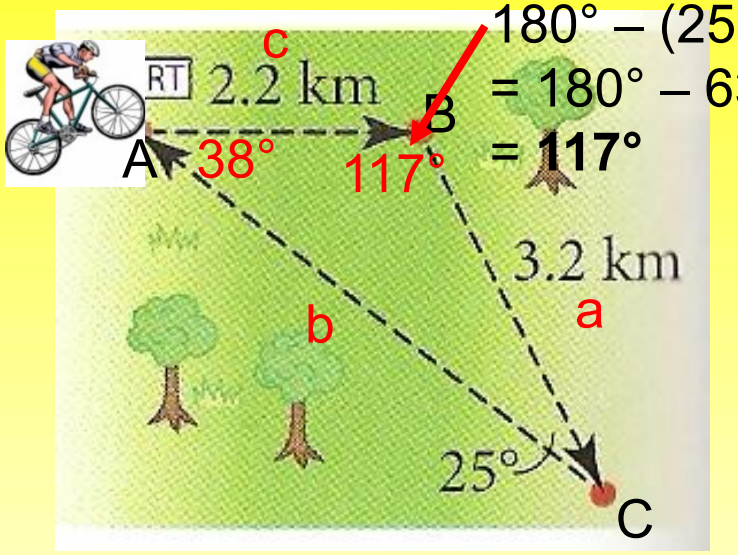
→Label the diagram

→Write down *Sine Law* formula and see what info we have!

CHAPTER 2.5 APPLICATIONS OF TRIGONOMETRY

EXAMPLE 2

Set a Bike Course



(a) What is the distance of the *third* leg of the bike route? Express your answer to the nearest tenth of a km.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{3.2}{\sin A} = \frac{b}{\sin B} = \frac{2.2}{\sin 25°}$$

Solve for ∠**A** first

$$\frac{3.2}{\sin A} = \frac{2.2}{\sin 25°}$$

$$\frac{2.2 \sin A}{2.2} = \frac{3.2 \sin 25°}{2.2}$$

$$\sin A = 0.6147$$

$$\angle A = \sin^{-1}(0.6147)$$

$$\angle A = 38°$$

We can use the **Sine Law**

→We have an angle and its opposite side

BUT....

We are missing *some* pieces of information

→Label the diagram

→Write down *Sine Law* formula and see what info we have!

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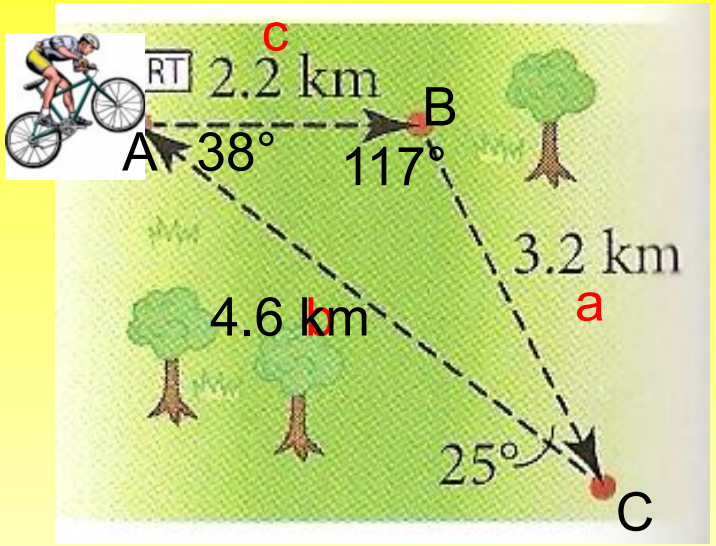
EXAMPLE 2

Set a Bike Course



(a) What is the distance of the *third* leg of the bike route? Express your answer to the nearest tenth of a km.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{3.2}{\sin 38°} = \frac{b}{\sin 117°} = \frac{2.2}{\sin 25°}$$

Solve for "b"

$$\frac{b}{\sin 117°} = \frac{2.2}{\sin 25°}$$

$$\frac{b \sin 25°}{\sin 25°} = \frac{2.2 \sin 117°}{\sin 25°}$$

$$b = 4.6\,km$$

The third leg of the race is **4.6 kilometres**

We can use the **Sine Law**

→We have an angle and its opposite side

BUT....

We are missing *some* pieces of information

→Label the diagram

→Write down *Sine Law* formula and see what info we have!

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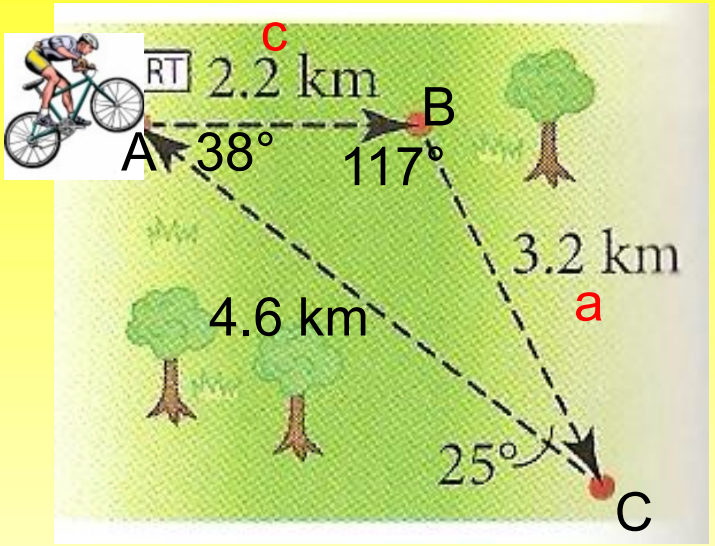
Set a Bike Course



(b) What is the total distance of <u>one</u> lap of the course?

<u>Add</u> up all legs of the race

= 2.2 + 3.2 + 4.6

*= **10 km***

The total distance of one lap of the course is **10 kilometres**

(c) How many laps are completed if the total bicycle ride is **40 km**?

$$\#\,of\ laps = \frac{Total\ distance\ of\ race}{Total\ distance\ of\ one\ lap}$$

$$\#\,of\ laps = \frac{40\,km}{10\,km}$$

$$\#\,of\ laps = 4\,laps$$

4 laps will be completed if the total bicycle ride is 40 km

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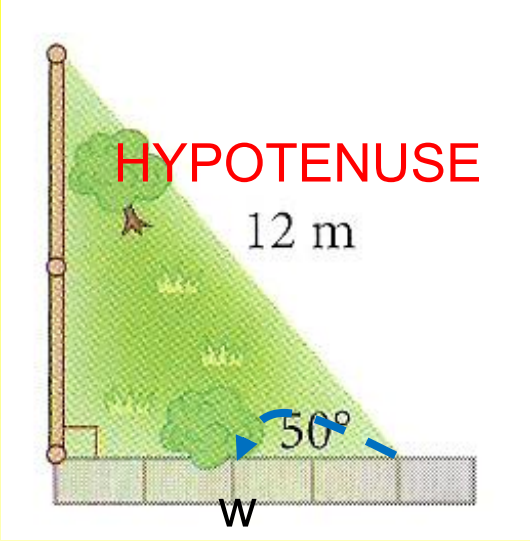
CHAPTER 2.5 APPLICATIONS OF TRIGONOMETRY

EXAMPLE 3 *Garden design*

Jonessa is making a new garden in the corner of her backyard. It will be against a privacy fence (on the left) and a patio walkway, as shown.



(a) How long is the **patio walkway**? Express your answer to the nearest metre.

→ Right-angle triangle

→ Use **basic trigonometric ratios**

Let "w" represent the length of the patio

$$\cos 50° = \frac{ADJACENT}{HYPOTENUSE}$$

Put over "1" → $\cos 50° = \frac{w}{12}$

$$12\cos 50° = w$$

$$7.7\,m = w$$

Cross multiply → $\frac{\cos 50°}{1} = \frac{w}{12}$

$$8\,m = w$$

The patio walkway is **8 metres**

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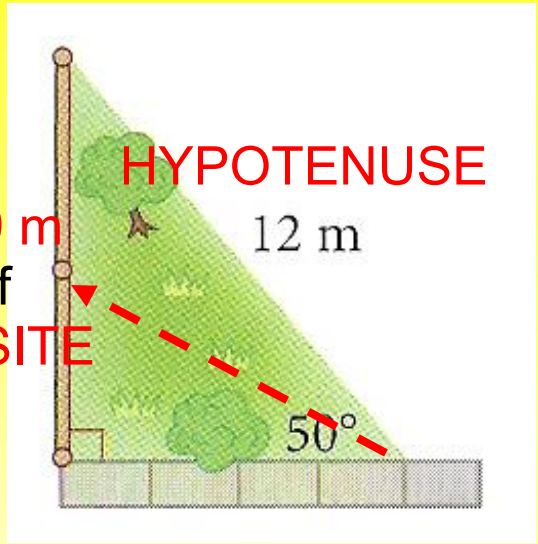

(b) How long is the **privacy fence**? Express your answer to the nearest metre.

Let "f" represent length of fence

$$\sin 50° = \frac{OPPOSITE}{HYPOTENUSE}$$

Put over "1" →

$$\sin 50° = \frac{f}{12}$$

$$12 \sin 50° = f$$

$$9.1m = f$$

Cross multiply →

$$\frac{\sin 50°}{1} = \frac{f}{12}$$

$$9m = f$$

The fence is 9 **metres**

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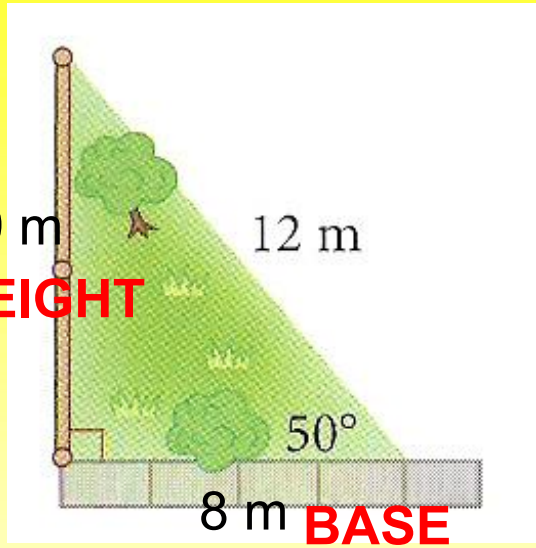

(c) How much fencing will Jonessa need to enclose the **entire garden** (except the fence)?

Add all lenghts (except fence)
= 8 + 12
= **20 m**

Jonessa will need **20 metres** of fencing

(d) Calculate the **total area** of the garden which will be enclosed.

$$A_{TRIANGLE} = \frac{base \times height}{2}$$

$$A_{TRIANGLE} = \frac{(8)(9)}{2}$$

$$A_{TRIANGLE} = 36m^2$$

Total area to be enclosed is **36 square metres**

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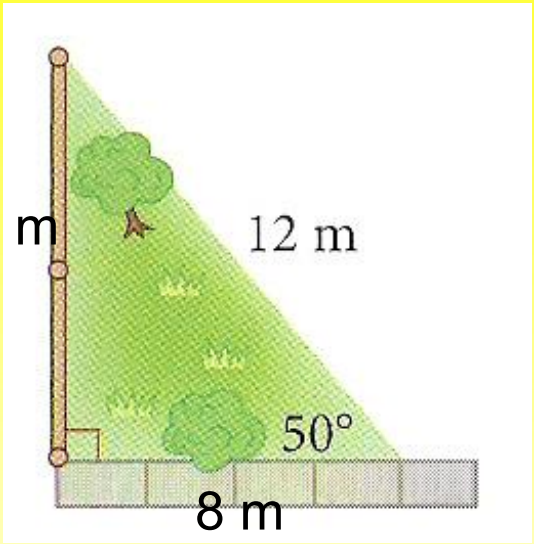

(e) If one bag of topsoil covers an **area of 4m²**, how many bags of topsoil will be required?

$$\#\,of\ bags = \frac{Total\ area}{Area\ covered\ by\ one\ bag}$$

$$\#\,of\ bags = \frac{36\,m^2}{4\,m^2}$$

$$\#\,of\ bags = 9$$

9 bags of topsoil will be required

(d) Calculate the **total area** of the garden which will be enclosed.

$$A_{TRIANGLE} = \frac{base \times height}{2}$$

$$A_{TRIANGLE} = \frac{(8)(9)}{2}$$

$$A_{TRIANGLE} = 36\,m^2$$

Total area to be enclosed is **36 square metres**

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HOMEWORK

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