ST. JEAN DE BREBEUF
MATHEMATICS

CHAPTER 2.4

THE COSINE LAW

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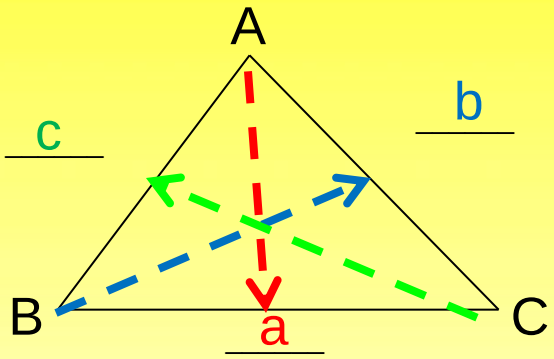
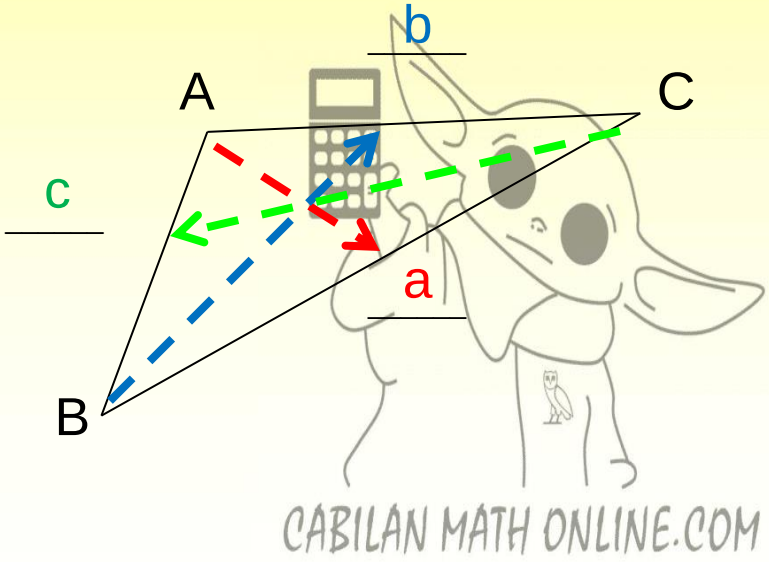
KEY CONCEPTS

The **Cosine Law** can be applied to both *acute* and *obtuse* triangles



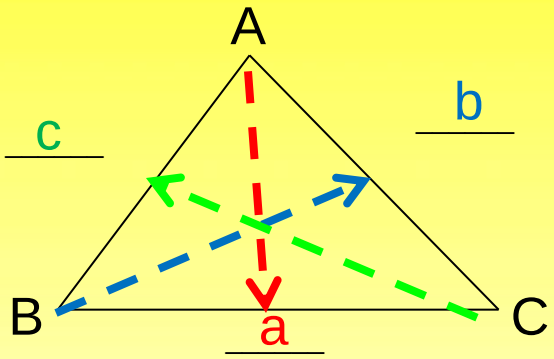
The formulas for any oblique triangle depend on the information given:



CHAPTER 2.4 THE COSINE LAW

KEY CONCEPTS

A **side length** can be determined if the other two sides and the *contained* angle are known
→known as **SIDE-ANGLE-SIDE** or **SAS**



Solving for **length of a side**

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

An **angle measure** can be determined if all three side lengths are given
→ known as **SIDE-SIDE-SIDE** or **SSS**

Solving for an **angle**

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

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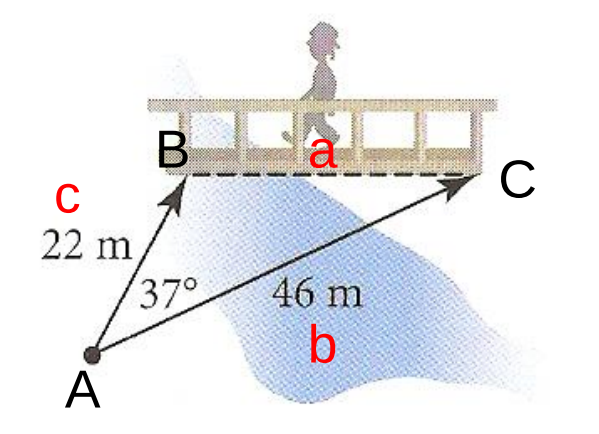
CHAPTER 2.4 THE COSINE LAW

EXAMPLE 1
Using the Cosine Law to Determine a Side Length

Hamid is a surveyor. He is on one side of a creek looking at both ends of a bridge. He measures the distances and angles shown in the diagram provided.

Determine the length of the bridge to nearest <u>tenth of a metre</u>.



•Label the triangle (corners and sides) first!!!

We can use the **Cosine Law** → **SIDE-ANGLE-SIDE (SAS)** is given

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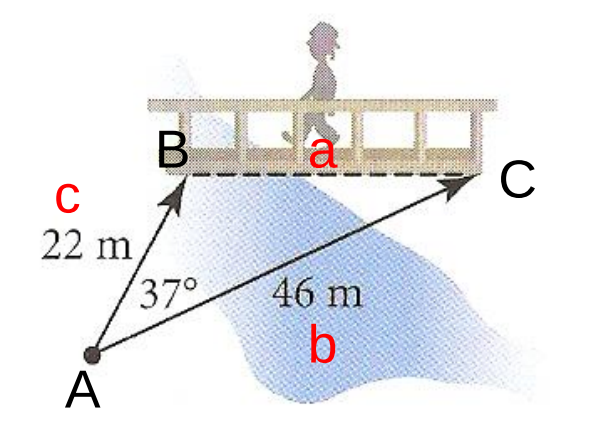


- Label the triangle (corners and sides) first!!!
- Write down formula and substitute known values

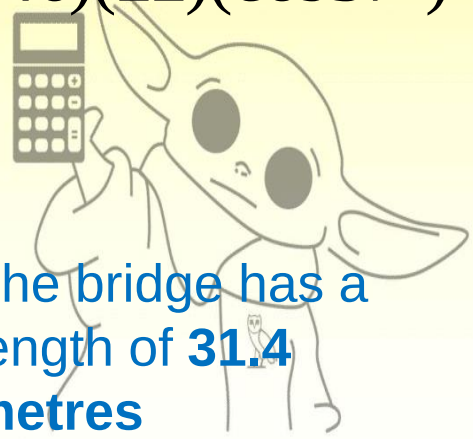
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = (46)^2 + (22)^2 - 2(46)(22)(\cos 37°)$$

$$a^2 = 983.5617$$

$$\sqrt{a^2} = \sqrt{983.5617}$$

$$a = 31.4m$$



The bridge has a length of **31.4 metres**

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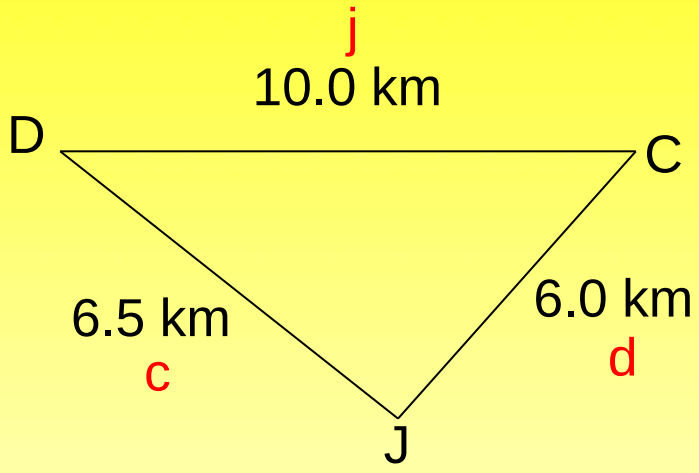
EXAMPLE 2

Using the Cosine Law to Solve for an Unknown Angle

Jada (J) is playing the role of a knight in a role playing style video game. Her battle helmet protects her from danger but restricts her field of view, which is the total angle in front of her around which she can clearly see.

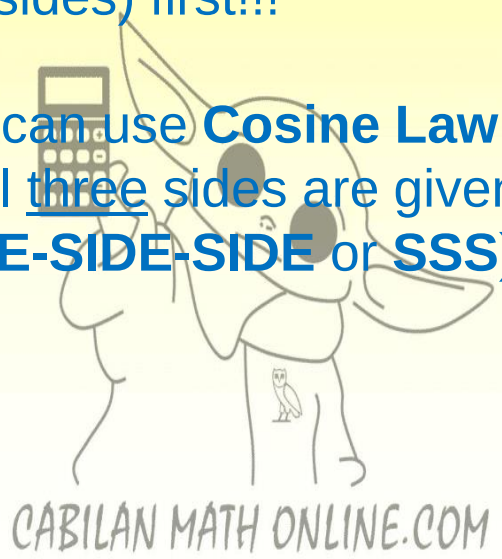
Jada can just see a castle (C) and a dragon (D) at the corners of her field of view. Jada estimates the distances, as shown in the simplified diagram provided.

Determine the angle of Jada's field of view and express your answer to the nearest degree.



- Label the triangle (corners and sides) first!!!

- We can use **Cosine Law**
→ All three sides are given
(SIDE-SIDE-SIDE or **SSS**)



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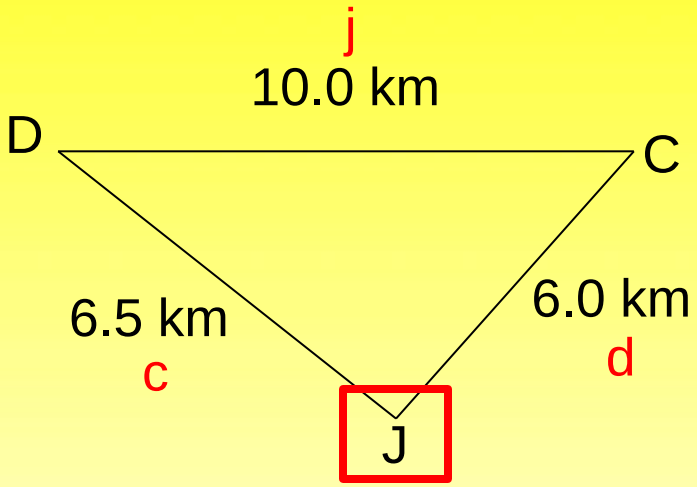
Determine the angle of Jada's field of view and express your answer to the nearest degree.



$$\cos J = \frac{c^2 + d^2 - j^2}{2cd}$$

$$\cos J = \frac{(6.5)^2 + (6.0)^2 - (10.0)^2}{2(6.5)(6.0)}$$

$$\cos J = \frac{42.25 + 36 - 100}{78}$$

$$\cos J = \frac{-21.75}{78}$$

$$\cos J = -0.2788$$

$$\angle J = \cos^{-1}(-0.2788)$$

$$\boxed{\angle J = 106°}$$

- Label the triangle (corners and sides) first!!!
- We can use **Cosine Law** → All three sides are given (**SIDE-SIDE-SIDE** or **SSS**)
- Write down formula and substitute known values

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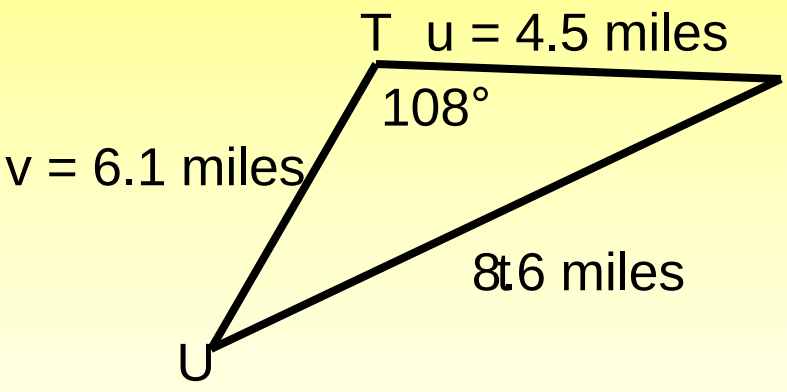
Jada's field of view is **106°**

CHAPTER 2.4 THE COSINE LAW

EXAMPLE 3 *Solve an Obtuse Triangle*

In ΔTUV, **u = 4.5 miles**, **v = 6.1 miles** and $\angle$**T = 108°**.

Solve ΔTUV. Round all *side lengths* to <u>one decimal place</u> and all *angles* to the <u>nearest degree</u>.



Solve for **"t"** by using **Cosine Law**
→ **SIDE-ANGLE-SIDE (SAS)** is given

$$t^2 = u^2 + v^2 - 2uv\cos T$$

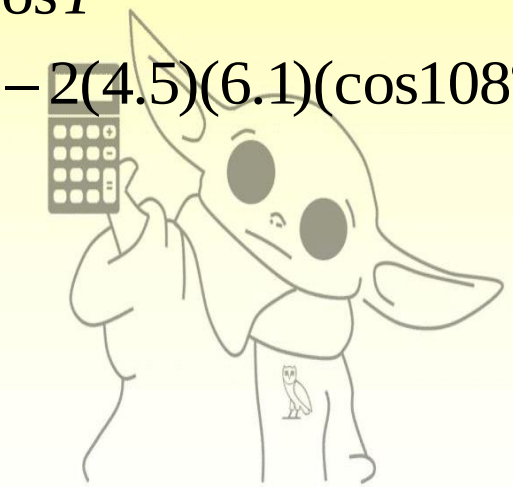
$$t^2 = (4.5)^2 + (6.1)^2 - 2(4.5)(6.1)(\cos 108°)$$

$$t^2 = 74.4250$$

$$\sqrt{t^2} = \sqrt{74.4250}$$

$$\boxed{t = 8.6}$$

- Draw an accurate diagram
- Label the triangle completely
- Write down formula and substitute known values



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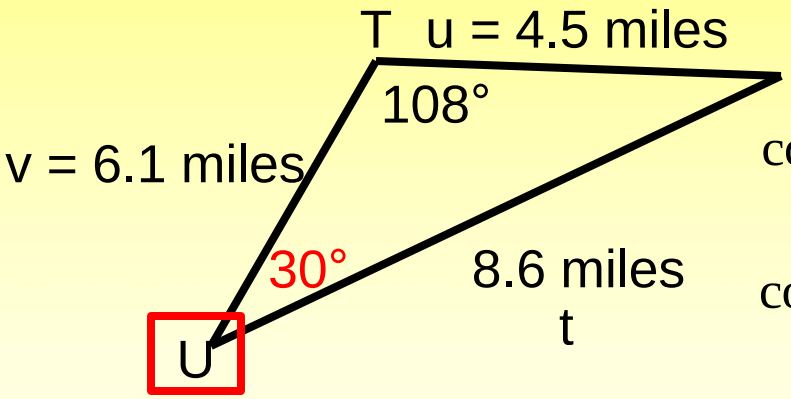


- Draw an accurate diagram
- Label the triangle completely
- Write down formula and substitute known values

Solve for ∠**U** by using **Cosine Law**

$$\cos U = \frac{t^2 + v^2 - u^2}{2tv}$$

$$\cos U = \frac{(8.6)^2 + (6.1)^2 - (4.5)^2}{2(8.6)(6.1)}$$

$$\cos U = \frac{73.96 + 37.21 - 20.25}{104.92}$$

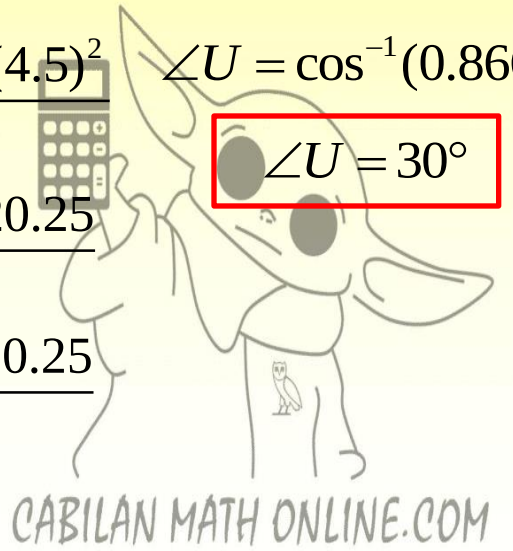
$$\cos U = \frac{73.96 + 37.21 - 20.25}{104.92}$$

$$\cos U = \frac{90.92}{104.92}$$

$$\cos U = 0.8666$$

$$\angle U = \cos^{-1}(0.8666)$$

$$\angle U = 30°$$



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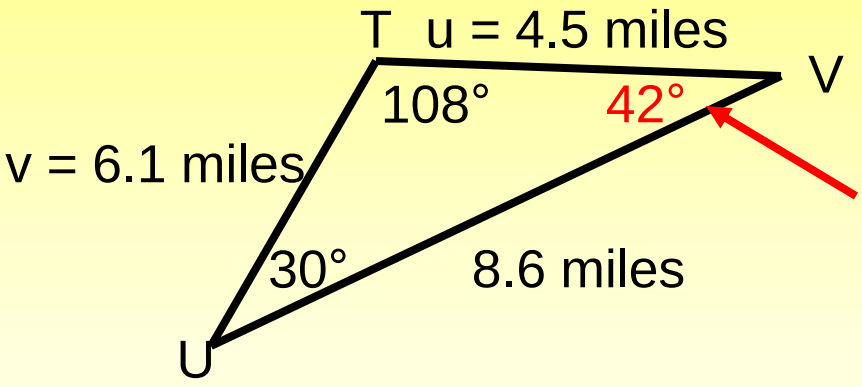

180° - (108 + 30)
= 180° - 138°
= **42°**

- Draw an accurate diagram
- Label the triangle completely
- Write down formula and substitute known values

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HOMEWORK

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