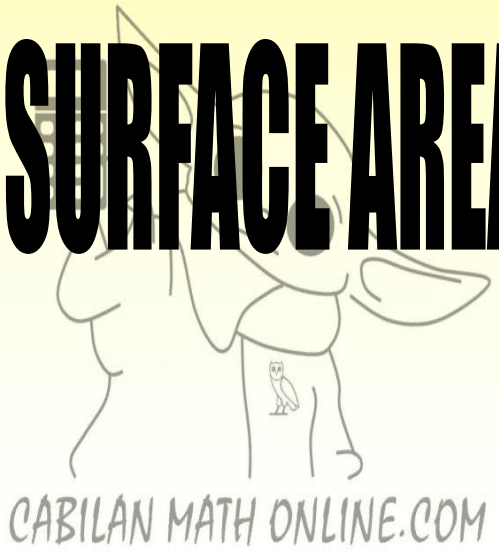
ST. JEAN DE BREBEUF
MATHEMATICS

CHAPTER 1.6 (PART 2)

ANALYSING OPTIMUM VOLUME and SURFACE AREA:

CYLINDERS

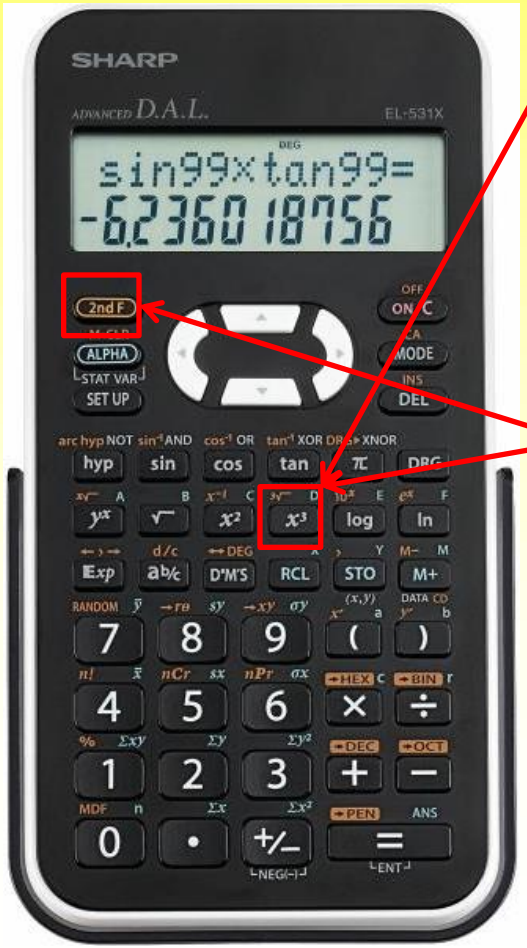


CHAPTER 1.6 (PART 2)

ANALYSING OPTIMUM VOLUME and SURFACE AREA: CYLINDERS

KEY CONCEPTS

USING YOUR CALCULATOR



x^3 button

→Raises a number to the exponent 3

EXAMPLE:

Evaluate **4^3**

→ Press "**4**" → **x^3** → **3** → **=**

2^{nd}F then x^3 button

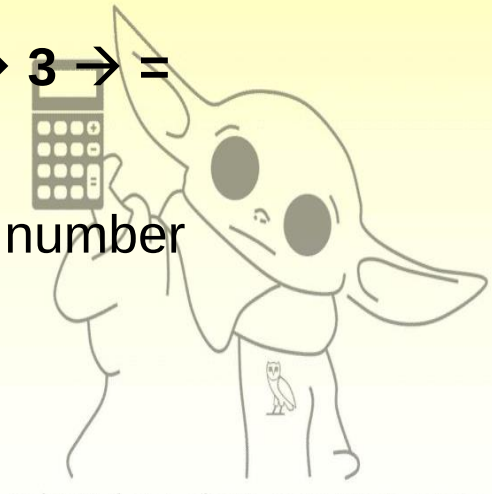
→*Cubed root* function

→Takes the *cubed root* of a number

EXAMPLE:

Evaluate $\sqrt[3]{125}$

→ Press **2ndF** → **x^3** → **125** → =



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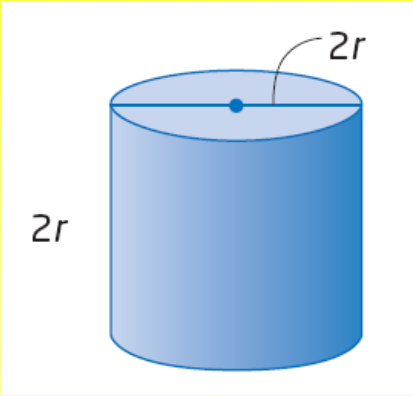

KEY CONCEPTS

For a **cylinder** with a given surface area, a **radius** and a **height** exist that produce the *maximum* volume.

The maximum volume for a given surface area of a cylinder occurs when its height equals its diameter.
→That is, $\boldsymbol{h = d}$ or $\boldsymbol{h = 2r}$.

The dimensions of the cylinder with maximum volume for a given surface area can be found by solving the formula $\boldsymbol{SA = 6\pi r^2}$ for *r*, and the **height** will be <u>twice</u> that value, or $\boldsymbol{2r}$.
→Once you solve for *r*, you can use the formula $\boldsymbol{V = \pi r^2 h}$ to solve for the volume (where $\boldsymbol{h = 2r}$)

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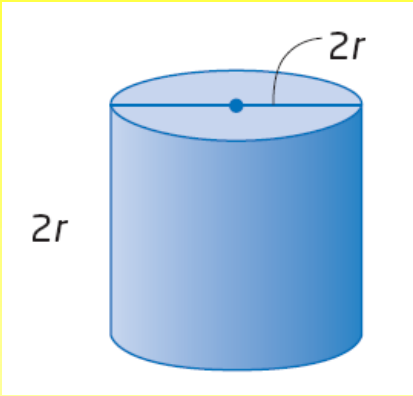
EXAMPLE



(a) Determine the dimensions of a *cylinder* with maximum volume if the **surface area is 1205.76 cm²**.

SA

$$SA = 6\pi r^2$$

$$\frac{1205.76}{6\pi} = \frac{6\pi r^2}{6\pi}$$

* Isolate **r²**
→Divide both sides by **6π**

$$64 = r^2$$

$$\sqrt{64} = \sqrt{r^2}$$

* Square root both sides

$$8\,cm = r$$

The *radius* of the cylinder will be **8 centimetres** and *height* will be **16 centimetres** (since h = 2r)

(b) What is the **volume** of this cylinder?

r = **8**

h = 2r
= 2(8)
= **16**

$$V = \pi r^2 h$$
$$= \pi(8)^2(16)$$
$$= 3215.36\,cm^3$$

Volume of the cylinder is **3215.35 cubic centimetres**.

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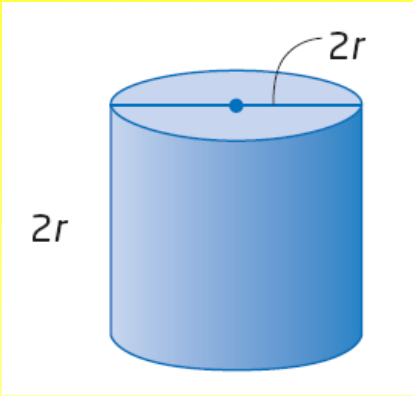
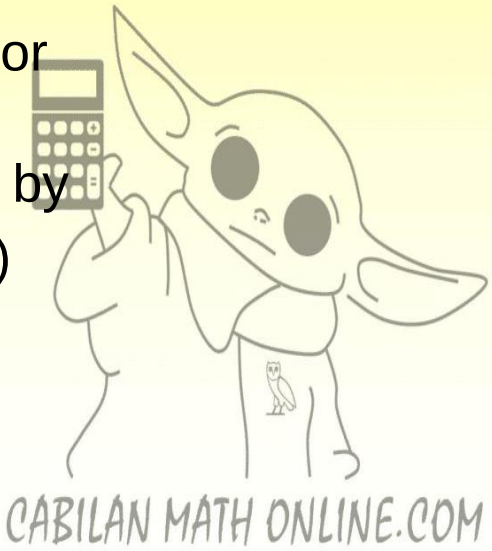


For a cylinder with a given volume, a radius and a height exist that produce the minimum surface area.

The minimum surface area for a given volume of a cylinder occurs when its height equals its diameter.
→That is, ***h = d*** or ***h = 2r***.

The dimensions of the cylinder of *minimum* surface area for a given volume can be found by solving the formula $\boldsymbol{V = 2\pi r^3}$ for ***r***, and the **height** will be *twice* that value, or **2*r***.
→Once you solve for *r*, you can find the **surface area** by using the formula $\mathbf{SA = 2\pi r^2 + 2\pi rh}$ (where **h = 2r**)



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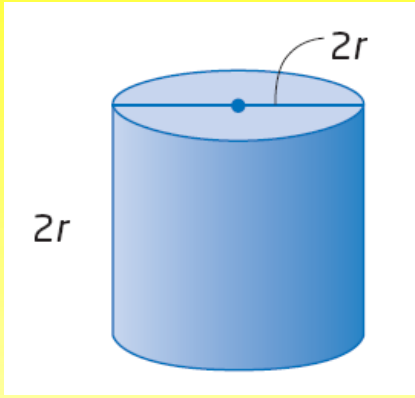
EXAMPLE



Determine the least amount of aluminum required to construct a cylindrical can with a **2.15405 litre** capacity (recall 1 L = 1000 cm^3).

Step 1: Convert **2.15405 litres** to **cm^3**.

* Need to multiply by **1000**

2.15405 × 1000
= **2154.05 cm^3**

Step 2: With the converted volume, solve for the **radius**

$$V = 2\pi r^3$$

$$\frac{2154.05}{2\pi} = \frac{2\pi r^3}{2\pi}$$

* Isolate r^3
→Divide both sides by **2π**

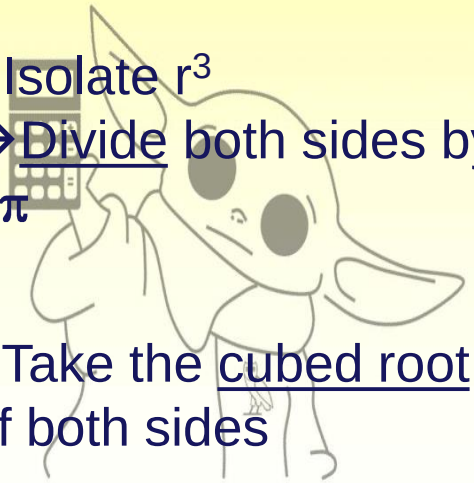
$$342.8277 = r^3$$

$$\sqrt[3]{342.8277} = \sqrt[3]{r^3}$$

* Take the cubed root of both sides

$$7\,cm = r$$

The *radius* of the cylinder is **7 cm** and *height* is **14 cm** (since **h = 2r**)



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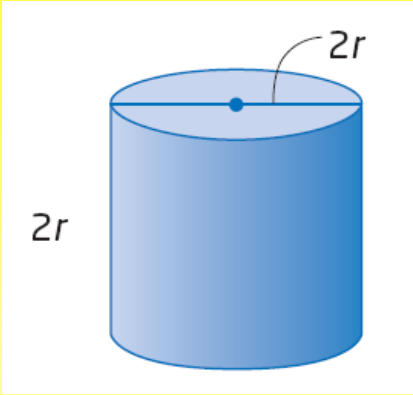
EXAMPLE



Determine the least amount of aluminum required to construct a cylindrical can with a **2.15405 litre** capacity (recall 1 L = 1000 cm^3).

The *radius* of the cylinder is **7 cm** and *height* is **14 cm** (since **h = 2r**)

Step 3: Solve for the **surface area** using the radius you found in *Step 2* (don't forget that **h = 2r**)

r = 7 h = 2r
= 2(7)
= **14**

$$SA = 2\pi r^2 + 2\pi rh$$
$$= 2\pi(7)^2 + 2\pi(7)(14)$$
$$= 923.63cm^2$$

The least amount of aluminum needed is **923.16 cm^2**.

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Homework:

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