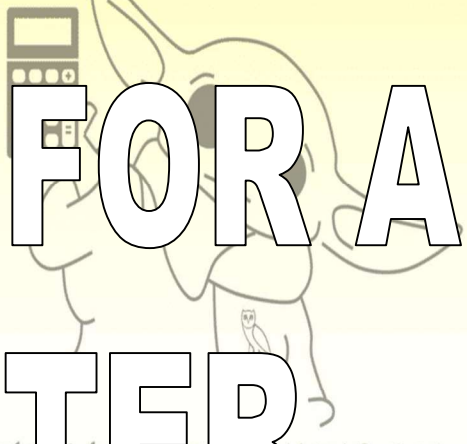
ST. JEAN DE BREBEUF
MATHEMATICS

CHAPTER 14 (PART 2)

MAXIMUM AREA FOR A GIVEN PERIMETER



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MAXIMUM AREA FOR A GIVEN PERIMETER

In the previous activity you explored how to **minimize** the *perimeter* of a rectangle given a fixed *area*.

In this activity you will discover how to **maximize** the *area* of a rectangle given a *fixed perimeter*.

Example

Students at St. Jean de Brebeuf want to create a prayer/meditation area outside the school's chapel. School council approves the idea and purchases a railing to enclose the area.

They purchase **18m** of railing that comes in **1m** sections that cannot be cut. The students want to build a rectangular prayer area with the largest possible area.

Determine the dimensions of the prayer area. If they want to carpet this space how much carpeting do they require?

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Example

In your notebook sketch the different rectangles that would fit these constraints. When you have completed your sketches, copy and fill in the table below.

PERIMETER
18 metres

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Example

In your notebook sketch the different rectangles that would fit these constraints. When you have completed your sketches, copy and fill in the table below.

$P = 2(L + W)$ $A = LW$

Length (m)	Width (m)	Perimeter (m)	Area (m^2)
8	1	18	8
7	2	18	A = (7)(2) = **14 m^2**
6	3	18	A = (6)(3) = **18 m^2**
5	4	18	A = (5)(4) = **20 m^2**

*Therefore, the dimensions must be **5m by 4m** with an Area of **20m^2**.*

→ Although not a square, it is pretty close!

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Did You Know?

If you could cut the railing and assemble it into a square you could improve the area of the prayer space?

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Length (m)	Width (m)	Perimeter (m)	Area (m^2)
4.9	4.1	18	20.09
4.8	4.2	18	20.16
4.7	4.3	18	20.21
4.6	4.4	18	20.24
4.5	4.5	18	20.25

Therefore, by making a square you can increase your area by $0.25m^2$.

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Here is a method for easy calculations of unknown dimensions when you are able to make a perfect square:

The *maximum* area occurs when the rectangle is a **square**.

Perimeter (P) = s + s + s + s
= **4s**

Substitute the fixed perimeter in for **P**.

Solve using basic algebra skills.

Example

What is the maximum area of a rectangle with a perimeter of **40m**?

P

Solution

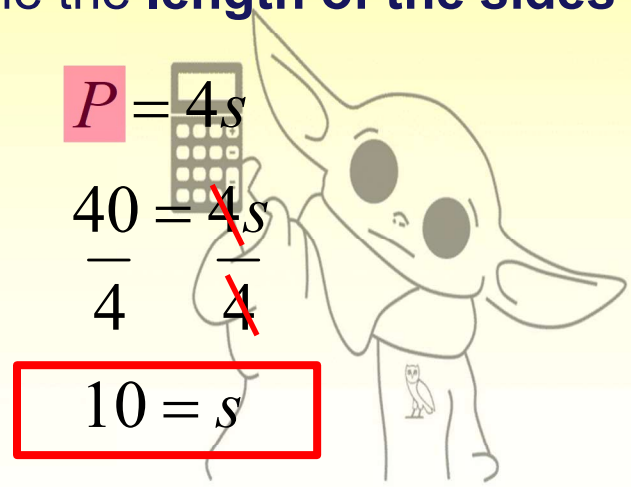
Step 1

Determine the **length of the sides**

$$P = 4s$$

$$\frac{40}{4} = \frac{4s}{4}$$

$$10 = s$$



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Step 2

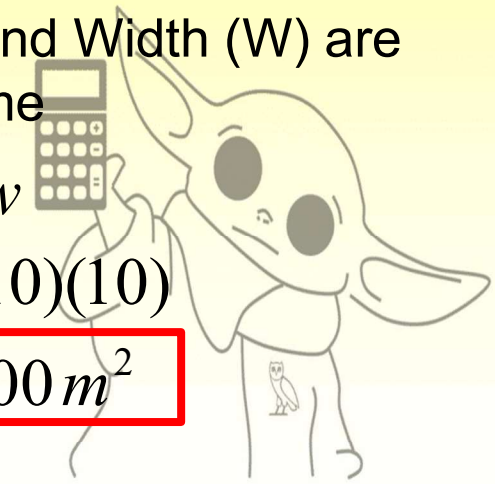
Find the **Area**

→ Length (L) and Width (W) are exactly the same

$$A = lw$$

$$A = (10)(10)$$

$$A = 100\,m^2$$



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*The maximum area is **100 m²**.*

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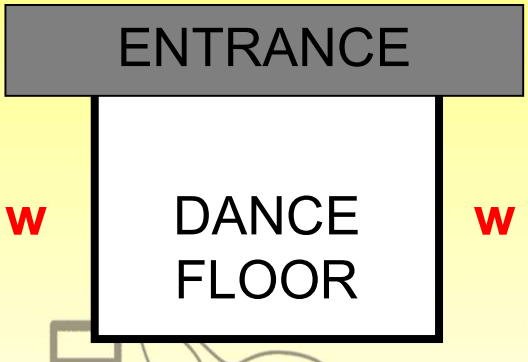
Example

What happens when a rectangle only has **three** sides?
→The maximum area for a given rectangular area is obtained by forming a rectangle where the **length is <u>twice</u>** its **width!!!**

A nightclub bouncer was given **260 metres** of velvet rope to section off the dance area from the main entrance, such that only *three* sides of the area are actually roped in (as pictured).



(i) Using the **perimeter**, determine the dimensions of the rectangle which would *maximize* the area

*** Remember that the **length is <u>twice</u> the width!**
→ l = 2w

Perimeter = sum of all sides
= w + l + w
= w + 2w + w
= **4w**

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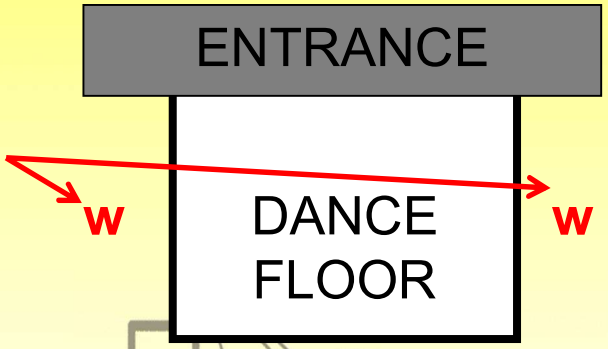


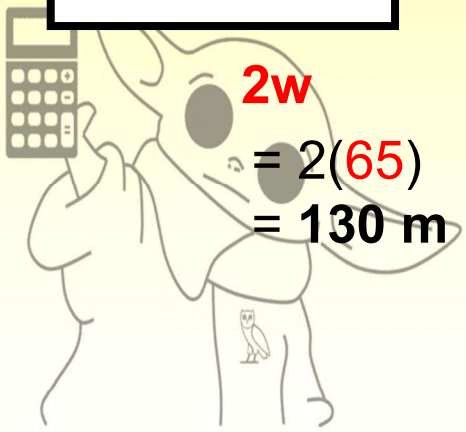
(i) Using the **perimeter**, determine the dimensions of the rectangle which would *maximize* the area
*** Remember that the **length is twice the width!**

Perimeter = **4w**

$$\frac{260}{4} = \frac{4w}{4}$$

65 = w

2w
= 2(65)
= **130 m**



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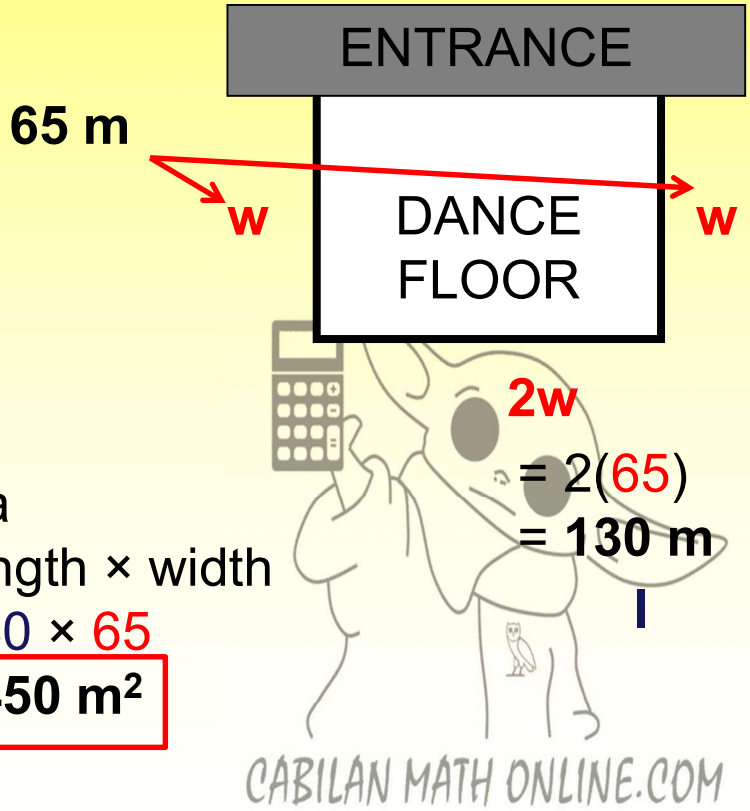

(ii) Using the dimensions from (i), calculate the **area**.

Area
= length × width
= 130 × 65
= **8450 m²**

The area of the dance floor is **8450 square metres**.

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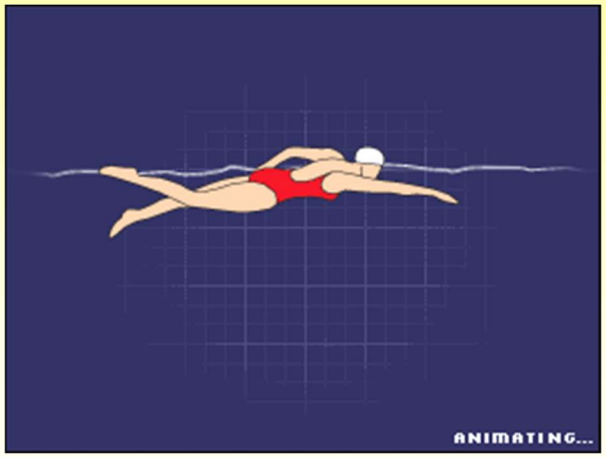
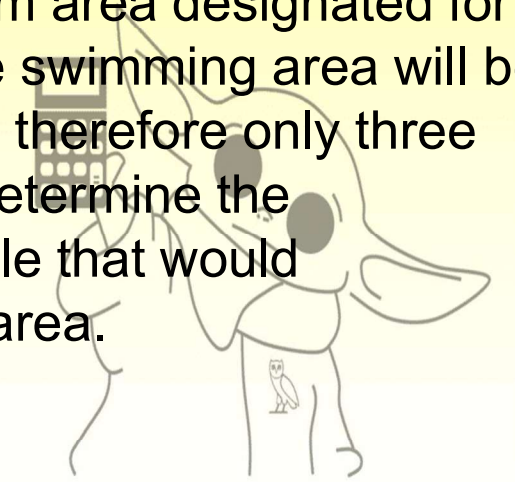
Assignment

1. A renovation company donates **80m** of fence to a day-care center to build a rectangular play area for the children. Determine the *dimension* of the play area that would produce the **largest** possible *area*? (Assume that the fence can be cut to suit measurements.)



2. A lifeguard has been given **500m** of rope to enclose a rectangular swim area designated for toddlers and children. The swimming area will be accessed from the beach; therefore only three sides must be roped in. Determine the *dimensions* of the rectangle that would **maximize** the swimming area.



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Assignment

3. Suppose you have **50** sections of fence to enclose a rectangular picnic area. Each piece of fence is **1 m** long and cannot be cut. Calculate the **maximum area** for the picnic area using the entire fence.

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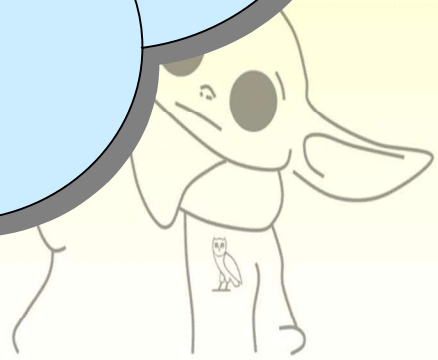
Homework

pg. 42 - 44 #6, 8, 12, 14



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