



ST. JEAN DE BREBEUF
MATHEMATICS

NUMBER SENSE

OPERATIONS with

RATIONAL NUMBERS

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

Rational number operations can often be simplified by converting **mixed numbers** to **improper fractions**

EXAMPLE: $2\frac{5}{7} = \frac{(7 \times 2) + 5}{7} = \frac{14 + 5}{7} = \frac{19}{7}$

Diagram description: The example shows the conversion of the mixed number 2 5/7 to the improper fraction 19/7. A blue arrow labeled 'Add' points from the numerator 5 to the product (7 x 2) + 5. A red arrow labeled 'Multiply' points from the whole number 2 to the product (7 x 2) + 5.

Negative fractions can be written with the negative number in the *numerator*

EXAMPLE: $-\frac{3}{4}$ can also be written as $\frac{-3}{4}$

To **add** or **subtract** rational number operations

→ Convert all mixed numbers to *improper fractions* (if required)

→ find a **common denominator** (a number that is divisible by *both* denominators)

→ Apply the **rules of integer operations** to the numerators and denominators

When **dividing** by a rational number, you have to **multiply** by its reciprocal

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

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WARM UP Adding and Subtracting Fractions

Evaluate the following and express in lowest terms if required:

$$\begin{aligned} \text{(a)} \quad & \frac{5}{8} + \frac{1}{2} \\ &= \frac{5(1) + 1(4)}{8} \\ &= \frac{5 + 4}{8} \end{aligned}$$

Common denominator
= 8

$$\boxed{= \frac{9}{8}}$$

$$\text{(b)} \quad \frac{7}{3} + \left(\frac{-1}{12} \right)$$

Common denominator = 12

$$= \frac{7}{3} - \frac{1}{12} = \frac{28 - 1}{12}$$
$$\boxed{= \frac{9}{4}}$$

$$= \frac{7(4) - 1(1)}{12} = \frac{27}{12}$$

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WARM UP Adding and Subtracting Fractions

Evaluate the following and express in lowest terms if required:

$$(c) \quad \frac{9}{4} - \frac{7}{20}$$

$$= \frac{9(5) - 7(1)}{20}$$

$$= \frac{45 - 7}{20}$$

Common denominator
= **20**

$$= \frac{38}{20}$$

$$= \frac{19}{10}$$

$$(d) \quad \frac{4}{10} - \left(\frac{-6}{15} \right)$$

$$= \frac{4}{10} + \frac{6}{15}$$

$$= \frac{4(3) + 6(2)}{30}$$

$$= \frac{12 + 12}{30}$$

Common denominator
= **30**

$$= \frac{24}{30}$$

$$= \frac{4}{5}$$

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EXAMPLE 1 Adding and Subtracting Mixed Numbers

Evaluate the following and express each answer in lowest terms. If the answer is an *improper fraction*, express the answer as a **mixed fraction**

$$\begin{aligned} \text{(a)} \quad & 2\frac{3}{5} + \frac{7}{10} &= \frac{13}{5} + \frac{7}{10} & \text{Common} \\ & & & \text{denominator} \\ & & & = \mathbf{10} \\ & = \frac{(5 \times 2) + 3}{5} + \frac{7}{10} &= \frac{13(2) + 7(1)}{10} &= \frac{33}{10} \\ & = \frac{10 + 3}{5} + \frac{7}{10} &= \frac{26 + 7}{10} &= 3\frac{3}{10} \end{aligned}$$

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

To **add** or **subtract** rational number operations

→ Convert all mixed numbers to *improper fractions* (if required)

→ find a **common denominator** (a number that is divisible by *both* denominators)

→ Apply the **rules of integer operations** to the numerators and denominators

EXAMPLE 1 Adding and Subtracting Mixed Numbers

Evaluate the following and express each answer in lowest terms. If the answer is an *improper fraction*, express the answer as a **mixed fraction**

$$\begin{aligned} \text{(b)} \quad 3\frac{1}{4} - \left(\frac{-5}{24}\right) &= \frac{12+1}{4} + \frac{5}{24} &= \frac{78+5}{24} \\ &= 3\frac{1}{4} + \frac{5}{24} & \text{Common denominator} &= \frac{83}{24} \\ &= \frac{(4 \times 3) + 1}{4} + \frac{5}{24} &= \mathbf{24} &= 3\frac{11}{24} \\ &= \frac{13(6) + 5(1)}{24} & & \end{aligned}$$

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

To **add** or **subtract** rational number operations

→ Convert all mixed numbers to *improper fractions* (if required)

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→ Apply the **rules of integer operations** to the numerators and denominators

EXAMPLE 1 Adding and Subtracting Mixed Numbers

Evaluate the following and express each answer in lowest terms. If the answer is an *improper fraction*, express the answer as a **mixed fraction**

$$\begin{aligned} \text{(c)} \quad & -4\frac{1}{6} - \frac{3}{8} & = -\frac{25}{6} - \frac{3}{8} & \text{Common} & = \frac{-109}{24} \\ & & & \text{denominator} & \\ & & & = \mathbf{24} & \\ & = -\frac{(6 \times 4) + 1}{6} - \frac{3}{8} & = \frac{-25(4) - 3(3)}{24} & & = -4\frac{13}{24} \\ & = -\frac{24 + 1}{6} - \frac{3}{8} & = \frac{-100 - 9}{24} & & \end{aligned}$$

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

EXAMPLE 2 *Multiplying Fractions*

When **multiplying** fractions

→ you multiply the numerators together

→ you multiply the denominators together

Evaluate the following by **multiplying** the fractions. Express each answer in lowest terms and express as a **mixed number** where required.

$$(a) \quad \frac{3}{7} \times \frac{2}{5}$$

$$= \frac{6}{35}$$

$$(b) \quad \left(\frac{-6}{11} \right) \left(\frac{-3}{5} \right)$$

$$= \frac{18}{55}$$

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EXAMPLE 2 *Multiplying Fractions*

When **multiplying** fractions

→ you multiply the numerators together

→ you multiply the denominators together

Evaluate the following by **multiplying** the fractions. Express each answer in lowest terms and express as a **mixed number** where required.

$$\begin{aligned} \text{(c)} \quad & \frac{-4}{9} \times \left(2\frac{5}{11} \right) \quad * \text{ Change to } & = \frac{-108}{99} \\ & \text{improper fraction} & \\ & = \frac{-4}{9} \times \left(\frac{(11 \times 2) + 5}{11} \right) & = -1\frac{9}{99} \\ & = \frac{-4}{9} \times \left(\frac{22 + 5}{11} \right) & = -1\frac{1}{11} \\ & = \frac{-4}{9} \times \frac{27}{11} & \end{aligned}$$

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

EXAMPLE 3 Dividing Fractions

When **dividing** by a rational number, you have to **multiply** by its reciprocal
→ You can find the reciprocal by inverting or “flipping” the fraction

Evaluate the following by **dividing** the fractions. Express each answer in lowest terms and express as a **mixed fraction** where required.

$$(a) \quad \frac{7}{8} \div \frac{4}{3}$$

$$= \frac{7}{8} \times \frac{3}{4}$$

$$= \frac{21}{32}$$

* Find the reciprocal by “flipping” the fraction

* Change from division to multiplication

$$(b) \quad 3\frac{4}{5} \div \left(\frac{-2}{9}\right)$$

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EXAMPLE 3 Dividing Fractions

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Evaluate the following by **dividing** the fractions. Express each answer in lowest terms and express as a **mixed fraction** where required.

$$(a) \quad \frac{7}{8} \div \frac{4}{3}$$

* Change to
improper fraction

$$= \frac{7 \curvearrowright 3}{8 \curvearrowright 4} \times \frac{3}{4}$$

$$= \frac{21}{32}$$

$$(b) \quad 3\frac{4}{5} \div \left(\frac{-2}{9}\right)$$

* Find the
reciprocal by
“flipping” the
fraction

$$= \frac{(5 \times 3) + 4}{5} \times \frac{9}{-2}$$

$$= \frac{15 + 4}{5} \times \frac{9}{-2}$$

$$= \frac{19 \curvearrowright 9}{5 \curvearrowright 2} \times \frac{9}{-2}$$

* Change from
division to
multiplication

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EXAMPLE 3 *Dividing Fractions*

When **dividing** by a rational number, you have to **multiply** by its reciprocal
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Evaluate the following by **dividing** the fractions. Express each answer in lowest terms and express as a **mixed fraction** where required.

$$(a) \quad \frac{7}{8} \div \frac{4}{3}$$

* Change to
improper fraction

$$= \frac{7 \overset{\curvearrowright}{\times} 3}{8 \underset{\curvearrowright}{\times} 4}$$

$$= \frac{21}{32}$$

$$(b) \quad 3\frac{4}{5} \div \left(\frac{-2}{9}\right)$$

$$= \frac{(5 \times 3) + 4}{5} \times \frac{9}{-2}$$

$$= \frac{15 + 4}{5} \times \frac{9}{-2}$$

$$= \frac{19 \overset{\curvearrowright}{\times} 9}{5 \underset{\curvearrowright}{\times} 2}$$

$$= \frac{171}{-10}$$

$$= -17\frac{1}{10}$$

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

EXAMPLE 3 Dividing Fractions

When **dividing** by a rational number, you have to **multiply** by its reciprocal
→ You can find the reciprocal by inverting or “flipping” the fraction

Evaluate the following by **dividing** the fractions. Express each answer in lowest terms and express as a **mixed fraction** where required.

$$\begin{aligned} \text{(c)} \quad & \left(-2\frac{1}{5}\right) \div \left(4\frac{2}{7}\right) \quad * \text{ Change to } \mathbf{\text{improper fraction}} \quad = -\frac{11}{5} \div \left(\frac{30}{7}\right) \\ & = -\frac{(5 \times 2) + 1}{5} \div \left(\frac{(7 \times 4) + 2}{7}\right) \quad = \frac{-11}{5} \times \frac{7}{30} \\ & = -\frac{10 + 1}{5} \div \left(\frac{28 + 2}{7}\right) \quad = \frac{-77}{150} \end{aligned}$$

* Find the *reciprocal* by “flipping” the fraction
* Change from *division* to *multiplication*

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

EXAMPLE 4 *Putting It All Together*

Evaluate the expression below and express the answer in lowest terms and express as a **mixed fraction** where required.

$$\boxed{5\frac{7}{11}} \div \left(\frac{3}{7} - \left(\frac{-1}{14} \right) \right)$$

* Change to **improper fraction**

$$= \frac{(11 \times 5) + 7}{11}$$

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

EXAMPLE 3 *Putting It All Together*

Evaluate the expression below and express the answer in lowest terms and express as a **mixed fraction** where required.

$$\boxed{5\frac{7}{11}} \div \left(\frac{3}{7} - \left(\frac{-1}{14} \right) \right) \quad * \text{ Change to improper fraction}$$

$$= \frac{55+7}{11}$$

NUMBER SENSE OPERATIONS with RATIONAL NUMBERS

EXAMPLE 3 Putting It All Together

Evaluate the expression below and express the answer in lowest terms and express as a **mixed fraction** where required.

$$5\frac{7}{11} \div \left(\frac{3}{7} - \frac{-1}{14} \right)$$

* Change to **improper fraction**

* Simplify terms in the brackets

$$= \frac{62}{11} \div \left(\frac{3}{7} + \frac{1}{14} \right)$$

Common denominator
= **14**

$$= \frac{62}{11} \div \left(\frac{3(2) + 1(1)}{14} \right)$$

$$= \frac{62}{11} \div \left(\frac{7}{14} \right)$$

* Find the *reciprocal* by “flipping” the fraction

* Change from *division* to *multiplication*

$$= \frac{62}{11} \times \frac{14}{7}$$

$$= \frac{868}{77}$$

$$= \frac{124}{11}$$

$$= 11\frac{3}{11}$$

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Seatwork/Homework:

**Attached Questions #3
and #5**