STEAN DE BREBEUF MATHEMATICS

CHAPTER 6.3

REPRESENT EXPONENTIAL EXPRESSIONS USING COMMON BASES

GET READY

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Solve for the variable 'x'

1. $x-3=17$

$x=17+3$

$x=20$

2. $-5+2x=-7$

$2x=-7+5$

$2x=-2$

$\frac{2x}{2}=\frac{-2}{2}$

$x=-1$

3. $6-5x=16$

$-5x=16-6$

$-5x=10$

$\frac{-5x}{-5}=\frac{10}{-5}$

$x=-2$

REPRESENT EXPONENTIAL EXPRESSIONSUSING COMMON BASES

KEY CONCEPTS

Powers can be represented in various ways, using different **base values**

If two equal powers have the same base, then their *exponents* must also be equal

It is sometimes useful to change the base of an exponential expression when solving equations

An equation in which the variable appears as an exponent or part of an exponent is called an **EXPONENTIAL EQUATION**

REPRESENT EXPONENTIAL EXPRESSIONSUSING COMMON BASES

EXAMPLE

$$2^x = 2^5$$

$$\boxed{x = 5}$$

Bases are already the same!

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on <u>both</u> sides of the equation the **same**
2. Use the **exponents** to make an equation
3. Solve

REPRESENT EXPONENTIAL EXPRESSIONSUSING COMMON BASES

EXAMPLE 1 *Solving for Variables when Bases are the SAME*

a) $2^x = 2^3$

$$x = 3$$

b) $5^{x+1} = 5^3$

$$x + 1 = 3$$

$$x = 3 - 1$$

$$x = 2$$

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on both sides of the equation the **same**
2. Use the **exponents** to make an equation
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PRACTICE

1. $4^{2x-1} = 4^7$

$$2x - 1 = 7$$

$$2x = 7 + 1$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

2. $5^{-x+7} = 5^{-5}$

$$-x + 7 = -5$$

$$-x = -5 - 7$$

$$-x = -12$$

-1

$$\frac{-x}{-1} = \frac{-12}{-1}$$

$$x = 12$$

REPRESENT EXPONENTIAL EXPRESSIONSUSING COMMON BASES

EXAMPLE 1 *Solving for Variables when Bases are the SAME*

PRACTICE

3. $3^{5x+2} = 3^{3x+12}$

$5x + 2 = 3x + 12$

*Gather variables on the **left side**

$5x - 3x + 2 = 12$

*Gather numbers on **right side**

$2x + 2 = 12$

$2x = 12 - 2$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on both sides of the equation the **same**
2. Use the **exponents** to make an equation
3. Solve

EXAMPLE 2 Changing to a Common Base

*Hint: The **base** is usually attached to the variable exponent*

$a) 2^x = 16$

Common base = 2

Express **16** as a base of **2**

→ $2^4 = 16$

$2^x = 2^4$

$\boxed{x = 4}$

$b) 4^{x-3} = 1$

Common base = 4

Express **1** as a base of **4**

→ $4^0 = 1$

$4^{x-3} = 4^0$

$x - 3 = 0$

$\boxed{x = 3}$

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on <u>both</u> sides of the equation the **same**
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EXAMPLE 2 *Changing to a Common Base*

*Hint: The **base** is usually attached to the variable exponent*

c) $3^{2x} = 81$

Common base = **3**

Express **81** with base **3**

→ $3^4 = 81$

$$3^{2x} = 3^4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$\boxed{x = 2}$$

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on <u>both</u> sides of the equation the **same**
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EXAMPLE 2 *Changing to a Common Base*

*Hint: The **base** is usually attached to the variable exponent*

PRACTICE

a) $5^{x-1} = 125$

Common base = **5**

Express **125** with a base of **5**

→ $5^3 = 125$

$$5^{x-1} = 5^3$$
$$x - 1 = 3$$
$$x = 3 + 1$$
$$\boxed{x = 4}$$

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on <u>both</u> sides of the equation the **same**
2. Use the **exponents** to make an equation
3. Solve

***b) $6^{x-1} = \frac{1}{216}$

Since this is a *fraction*, the exponent will be <u>negative</u>

Common base = **6**

→ $6^3 = 216$

→ $6^{-3} = 1 / 216$

$$6^{x-1} = 6^{-3}$$
$$x - 1 = -3$$
$$x = -3 + 1$$
$$\boxed{x = -2}$$

EXAMPLE 3 *Isolate the Power, Use Common Base and Solve*

$a)\ 4^{x+5} - 5 = 59$ Common base = **4**

$4^{x+5} = 59 + 5$

$4^{x+5} = 64$ ← Express **64** with a base of **4**

$4^{x+5} = 4^3$ → $4^3 = 64$

$x + 5 = 3$

$x = 3 - 5$

$\boxed{x = -2}$

TO SOLVE AN EXPONENTIAL EQUATION

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EXAMPLE 3 *Isolate the Power, Use Common Base and Solve*

$b)\ 250(2^n) = 8000$ Common base = **2**

$$\frac{250(2^n)}{250} = \frac{8000}{250}$$

$$2^n = 32$$ Express **32** with a base of **2**

$$2^n = 2^5$$ → $2^5 = 32$

$$\boxed{n = 5}$$

TO SOLVE AN EXPONENTIAL EQUATION

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EXAMPLE 3 *Isolate the Power, Use Common Base and Solve*

PRACTICE

a) $5^{x+10} = \frac{1}{5^x}$

Common base = **5**

Since this is a *fraction*, express this with a negative exponent

→ $5^{-x} = 1 / 5^x$

$$5^{x+10} = 5^{-x}$$

$$x + 10 = -x$$

*Gather variables on the **left**

$$x + x + 10 = 0$$

$$2x + 10 = 0$$

*Gather numbers on the **right**

$$\frac{2x}{2} = \frac{-10}{2}$$

$$x = -5$$

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on both sides of the equation the **same**
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EXAMPLE 3 *Isolate the Power, Use Common Base and Solve*

PRACTICE

***b) $8^{x} = 4^{-3}$ Common base = 2

2^3 2^2

$$\left(2^{3}\right)^{x} = \left(2^{2}\right)^{-3}$$ *Multiply exponents

$$2^{3x} = 2^{2\times(-3)}$$

$$2^{3x} = 2^{-6}$$

$$\frac{3x}{3} = \frac{-6}{3}$$

$$x = -2$$

TO SOLVE AN EXPONENTIAL EQUATION

1. Make the bases on both sides of the equation the **same**
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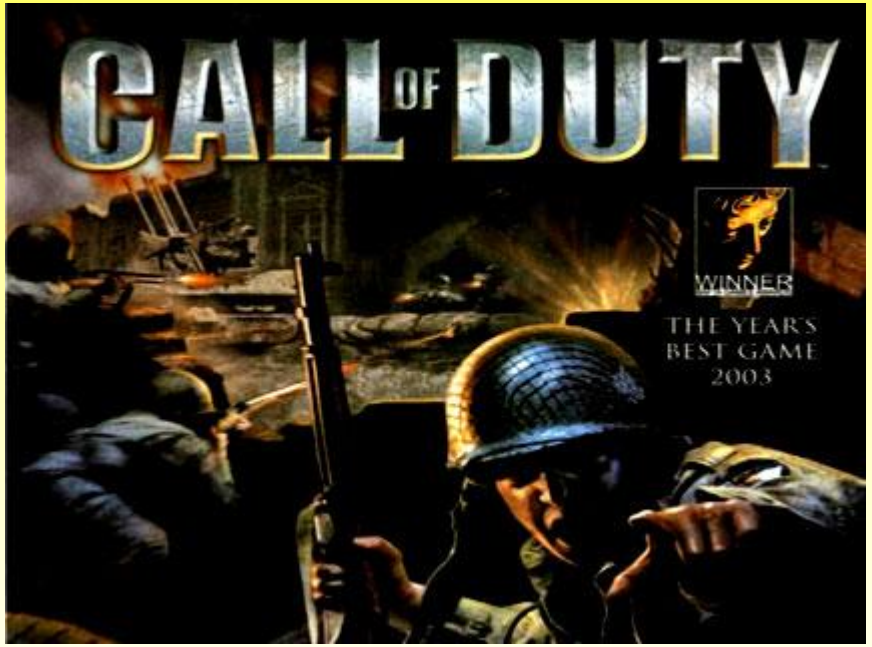
EXAMPLE 4
Application of Solving Exponential Expressions

Raj started playing *Call of Duty* on the Internet. As he played, he found that his score, *S*, *doubled* after every day, *d*. The relationship was represented by the equation $S = 2^d$.



Raj's best friend, Harvinder also started playing the game. To outscore Raj, Harvinder played the game so much that he *quadrupled* his score (S) after every day (d). The relationship was represented by the equation $S = 4^{d-3}$.

Using algebraic methods, determine after how many days Raj and Harvinder's scores will be the same.

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Application of Solving Exponential Expressions

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Raj's best friend, Harvinder also started playing the game. To outscore Raj, Harvinder played the game so much that he *quadrupled* his score (S) after every day (d). The relationship was represented by the equation **S = 4^{d-3}**.

Using algebraic methods, determine after how many days Raj and Harvinder's scores will be the same.

Raj's score: $S = 2^d$ **(1)**
Harvinder's score: $S = 4^{d-3}$ **(2)**

Perform substitution
→ Equate equations (1) and (2) to each other

$$2^d = 4^{d-3}$$

Common base = **2**
Express 4 as a base of **2**
→ $4 = 2^2$

$$2^d = (2^2)^{d-3}$$

$$2^d = 2^{2d-6}$$

$$d = 2d - 6$$

* Move variables to the left
* Numbers to the right

$$d - 2d = -6$$

$$\frac{-d}{-1} = \frac{-6}{-1}$$

$$d = 6$$

Raj's and Harvinder's score will be the same after **6 days**

HOMEWORK

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#1, 2, 4, 5, 6acf, 8a – d, 10ab