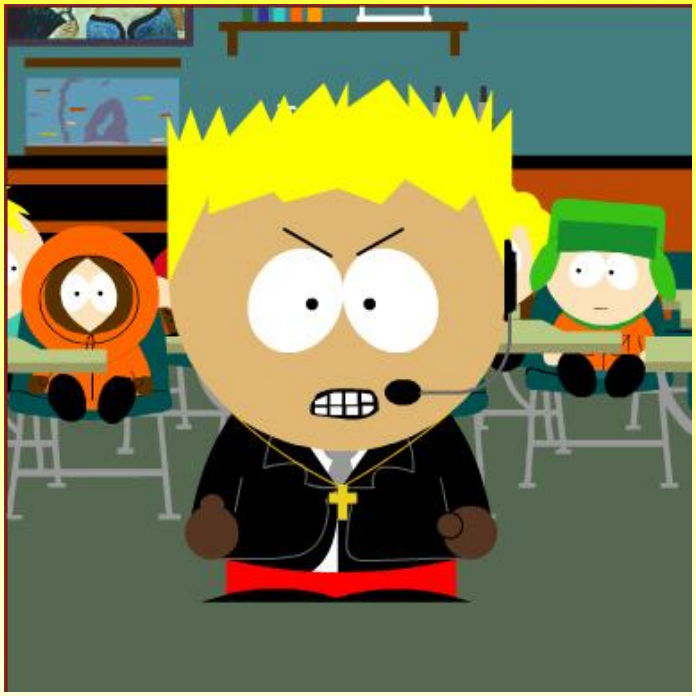
ST. JEAN DE BREBEUF MATHEMATICS



CHAPTER 6.2
RATIONAL EXPONENTS

RATIONAL EXPONENTS

KEY CONCEPTS

Exponents that are unit fractions represent **roots**

Square root $a^{\frac{1}{2}} = \sqrt{a}$

Cubed root $a^{\frac{1}{3}} = \sqrt[3]{a}$

$a^{\frac{1}{4}} = \sqrt[4]{a}$ ..... $a^{\frac{1}{n}} = \sqrt[n]{a}$

Fractional exponents, where the numerator is not 1 can be re-arranged:

$$b^{-1/2} = \frac{1}{b^{1/2}} = \frac{1}{\sqrt{b}} \qquad b^{-1/3} = \frac{1}{b^{1/3}} = \frac{1}{\sqrt[3]{b}} \text{ , ...} \quad b^{-1/n} = \frac{1}{b^{1/n}} = \frac{1}{\sqrt[n]{b}}$$

$$b^{3/2} = (b^{1/2})^3 \text{ or } (\sqrt{b})^3 \rightarrow b^{x/y} = (b^{1/y})^x \text{ or } (\sqrt[y]{b})^x$$

RATIONAL EXPONENTS

MEANING OF EXPONENTS ½ and 1/3

DISCOVER 1

1. What is the value of $\sqrt{9}$? 3

2. Use your answer in (1) to determine the value of $\sqrt{9} \times \sqrt{9}$

   = 3 x 3

   = 9

3. Use *Exponent Laws* that you learned in the previous lesson to determine the value of $9^{\frac{1}{2}} \times 9^{\frac{1}{2}}$.

$$= 9^{\frac{1}{2}+\frac{1}{2}}$$

$$= 9^{\frac{2}{2}}$$

$$= 9^{1}$$

$$= 9$$

RATIONAL EXPONENTS

DISCOVER 1

4. Looking at (2) and (3), what operation does the exponent $\frac{1}{2}$ seem to be the same as? Square root

IN GENERAL:

$\sqrt{a}$ can also be written as:

$$a^{1/2}$$

RATIONAL EXPONENTS

DISCOVER 2

The value of 2^3 is 8. Because of this, we can say the "cube root of 8 equals 2" and we can write this as $\sqrt[3]{8} = 2$

1. Given that $\sqrt[3]{8} = 2$, find the value for $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8}$.

   = 2 x 2 x 2

   = 8

2. Using the *Exponent Law* for Multiplication of like bases, find the value for $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}}$

$$= 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \qquad = 8^1$$

$$= 8^{\frac{3}{3}} \qquad \boxed{= 8}$$

RATIONAL EXPONENTS

3. Using your answer in (1) and (2), what operation does the exponent $\frac{1}{3}$ seem to be the same as? Cubed root

IN GENERAL:

$\sqrt[3]{a}$ can be written as:

$a^{1/3}$

RATIONAL EXPONENTS

IN GENERAL:

$\sqrt{a}$ can also be written as:

$$a^{1/2}$$

IN GENERAL:

$\sqrt[3]{a}$ can be written as:

$$a^{1/3}$$

EXAMPLE 1 *Evaluating Powers With a Rational Exponent of the form 1 / n*

a) $16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

b) $27^{\frac{1}{3}}$

$= \sqrt[3]{27}$

$= 3$

Radical form

RATIONAL EXPONENTS

IN GENERAL:

$\sqrt{a}$ can also be written as:

$a^{1/2}$

IN GENERAL:

$\sqrt[3]{a}$ can be written as:

$a^{1/3}$

EXAMPLE 1 *Evaluating Powers With a Rational Exponent of the form 1 / n*

c) $64^{-\frac{1}{3}}$ ← *Negative exponent

→"Flip" the base

→Make exponent **positive**

$\equiv \left(\frac{1}{64}\right)^{\frac{1}{3}}$

$= \frac{(1)^{\frac{1}{3}}}{(64)^{\frac{1}{3}}}$ *Raise numerator and denominator by the exponent

$= \frac{1}{\sqrt[3]{64}}$

$= \frac{1}{4}$

** (d) $(-16)^{\frac{1}{4}}$

$= \sqrt[4]{-16}$

Cannot evaluate!!!

RATIONAL EXPONENTS

EXAMPLE 2 *Evaluating Powers with Rational Exponent of the Form m / n*

Simplify $125^{2/3}$

$$125^{\frac{2}{3}}$$

$$= \left(125^{\frac{1}{3}}\right)^2$$

$$= \left(\sqrt[3]{125}\right)^2$$

$$= (5)^2$$

$$= 25$$

$$a^{\frac{m}{m}} = \left(a^{\frac{1}{n}}\right)^m$$

STEPS:

1. "Remove" the numerator (top number of the fraction) and place it <u>outside</u> a set of brackets

2. Re-write the *power* <u>inside</u> of the brackets where the fraction of the exponent has a numerator of **1**

3. Write the power in **radical** form and use your calculator to evaluate the radical

4. Simplify

RATIONAL EXPONENTS

EXAMPLE 2 *Evaluating Powers with Rational Exponent of the Form m / n*

b) $16^{\frac{3}{2}}$

$$= \left(16^{\frac{1}{2}}\right)^3$$

$$= \left(\sqrt[2]{16}\right)^3$$

$$= (4)^3$$

$$= 64$$

$$a^{\frac{m}{m}} = \left(a^{\frac{1}{n}}\right)^m$$

STEPS:

1. "Remove" the numerator (top number of the fraction) and place it <u>outside</u> a set of brackets

2. Re-write the *power* <u>inside</u> of the brackets where the fraction of the exponent has a numerator of **1**

3. Write the power in **radical** form and use your calculator to evaluate the radical

4. Simplify

RATIONAL EXPONENTS

EXAMPLE 2 *Evaluating Powers with Rational Exponent of the Form m / n*

c) $8^{-\frac{2}{3}}$ ← *Negative exponent

→"Flip" the base

→Make exponent **positive**

$\equiv \left(\frac{1}{8}\right)^{\frac{2}{3}}$

$= \frac{(1)^{\frac{2}{3}}}{(8)^{\frac{2}{3}}}$ *Raise numerator and denominator by the exponent

$= \frac{1}{\left(8^{\frac{1}{3}}\right)^2}$

$= \frac{1}{\left(\sqrt[3]{8}\right)^2}$

$= \frac{1}{(2)^2}$

$= \frac{1}{4}$

$$a^{\frac{m}{m}} = \left(a^{\frac{1}{n}}\right)^m$$

STEPS:

1. "Remove" the numerator (top number of the fraction) and place it <u>outside</u> a set of brackets
2. Re-write the *power* <u>inside</u> of the brackets where the fraction of the exponent has a numerator of **1**
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RATIONAL EXPONENTS

EXAMPLE 3

Applying Rational Exponents

A water tower supports a spherical holding tank with a volume of 500 m^3. The radius of the tank, r, can be determined by the formula

$$r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$$

where V is the volume.

Determine the *radius* of the tank to the nearest tenth of a metre.

The radius of the tank is **4.9 metres**

$$r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$$

$$r = \left(\frac{3(500)}{4\pi}\right)^{\frac{1}{3}}$$

$$r = \left(\frac{1500}{4\pi}\right)^{\frac{1}{3}}$$

$$r = (119.3662)^{\frac{1}{3}}$$

$$r = \sqrt[3]{119.3662}$$

* Express in *radical form*

$$\boxed{r = 4.9}$$

HOMEWORK

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#1, 3 – 5, 6abc, 7ace, 9, 11, 13ab